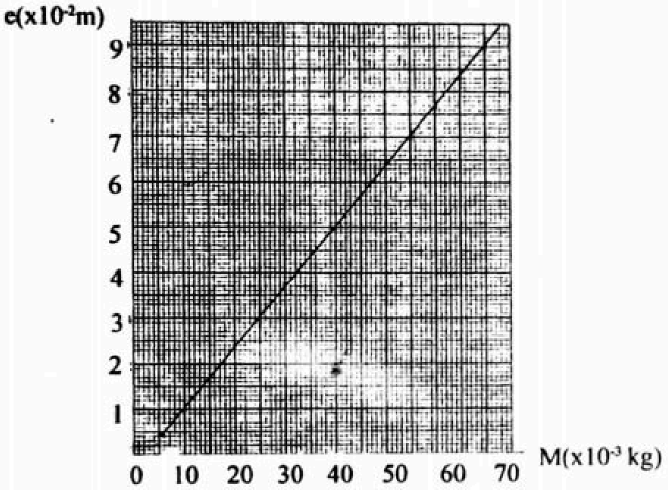
G.C.E. (Advanced Level) Examination - August 2008
PHYSICS - II
Three hours

Answer all four questions. **PART A - Structured Essay** $[g = 10\ N\ kg^{-1}]$

01. Figure shows a spring whose upper end is firmly clamped to a rigid stand, and a light pointer attached to its lower end. You are to determine the spring constant (k) of the spring. A set of standard weights and a metre ruler are provided.

(a) Draw the metre ruler on the figure at its correct poition in order to measure the extension (e) of the spring.

(b) The extension (e) versus load (M) graph for such a spring is shown below.



(i) Determine the spring constant k of the spring in $k\ gm^{-1}$.

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(ii) Clearly indicate on the graph the two points which you have used to determine k.

(c) The spring with a load M attached is set in vertical oscillations by giving a small displacement. The period (T) of the oscillations is given by

$$T = 2\pi\sqrt{\frac{M + \frac{m}{3}}{kg}}$$ where m is the mass of the spring.

(i) Rearrange the above expression in the **most suitable** manner to draw a graph in order to determine the acceleration due to gravity (g) and mass (m) of the spring.

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(ii) What additional instrument do you need to take measurements in this experiment?

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(iii) What quantities would you extract from the graph to determine g and m?

To determine g : ....

To determine m : ....

(d) If the percentage error of M values is 1%, how many oscillations would you need to take in order to match the percentage error of T also to 1%? (Fractional error of T is $\frac{2\Delta T}{T}$, and error in time measurement is 0.1s. Take $T = 2s$).

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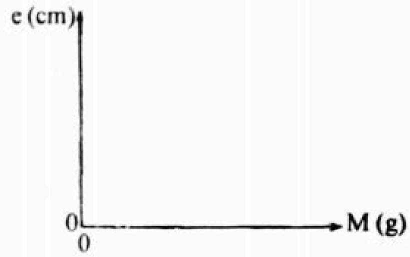
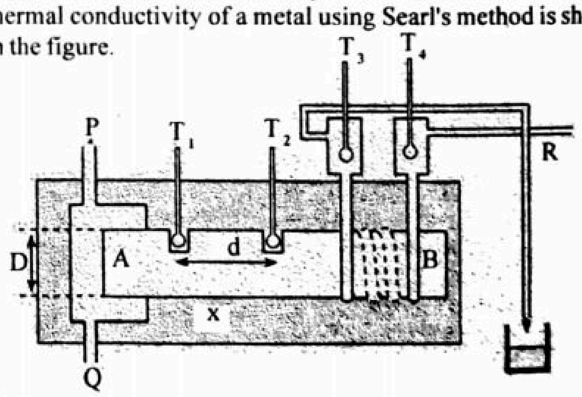
(e) A student used a brand new spring whose turns **are pressed against each other** to draw the graph mentioned in (*b*) above. On the Figure given below, sketch the shape of the graph that you would expect in this situation.



02. A part of an experimental setup that is used to determine the thermal conductivity of a metal using Searl's method is shown in the figure.



(a) Draw a figure of the apparatus that you need to connect to tube R, in the **appropriate place** of the space in front of R. Clearly show haw you would connect the apparatus to R.

(b) What additional instruments are essential in order to perform this experiment?

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(c) End A of the metal bar is heated using steam. Give **two** reasons as to why it is better to send steam through tube P rather than sending through tube Q.

(i) ....

(ii) ....

(d) How do you observe whether the system has reached the steady state?

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(e) How do you achieve a good thermal contact between thermometers T_1, T_2 and the metal bar?

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(f) You are provided with the following set of date related to this experiment.

Reading of the thermometer	T_1 (θ_1)	= 75.0°C
Reading of the thermometer	T_2 (θ_2)	= 61.0 °C
Reading of the thermometer	T_3 (θ_3)	= 37.0 °C
Reading of the thermometer	T_4 (θ_4)	= 28.0 °C
Mass of water collected in 3.0 minutes (M)		= 0.4kg
Cross sectional area of the metal rod (A)		= 1.2×10^{-3} m²
Distance between the thermometers T_1 and T_2 (d) = 0.08 m		
Specific heat capacity of water (s)		= 4200J kg^{-1} K^{-1}

Calculate the thermal conductivity of metal.

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(g) Space X is filled with a good thermal insulator such as polystyrene to reduce the heat loss from the metal bar. Thermal conductivity of air is 0.025Wm^{-1}K^{-1} and that of polystyrene is 0.08 Wm^{-1} K^{-1} which implies that air is a good thermal insulator than polystyrene. Explain why it is still better to fill the space X with polystyrene rather than having air.

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03. (a) Two monochromatic rays of light with angles of incidence $\theta_1(>\theta_C)$ and θ_2 ($<\theta_C$), where θ_C is the critical angle of glass, are falling on a glass-air interface as shown in figure 1. Complete the paths of the rays.

Figure 1

(b) You are asked to determine the critical angle of glass by the method of total internal reflection. A prism is placed on a whits sheet of paper, in such a way that a vertical pin (M) is in contact with face AC of the prism as shown in figure 2. The boundaries of the faces of the prism are drawn on the paper.

(i) In this experiment the pin M has to be placed in contact with face AC. State the reason for this.

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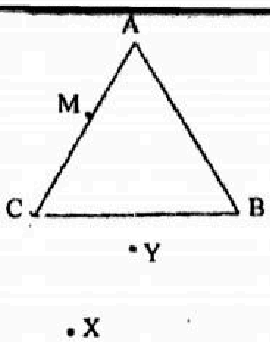

Figure 2

(ii) When you move your eye from B to C while looking through face BC towards AB what change of the image of the pin M do you expect to observe?

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(iii) How do you detect path of the relevant emergent ray experimentally using two other pins? The locations of the two pins are marked as X and Y in the figure 2.

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(iv) Write down the remaining steps in the order that you would follow to construct ray diagram.
Use figure 2 also to illustrate the steps in the construction of the ray diagram.

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(v) What measurement would you take from the ray diagram? Also clearly indicate it on the ray diagram.

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(c) You are asked to modify and repeat this experiment to determine the critical angle for glass - water interface by forming a thin layer of water on the surface AB as shown in figure 3.

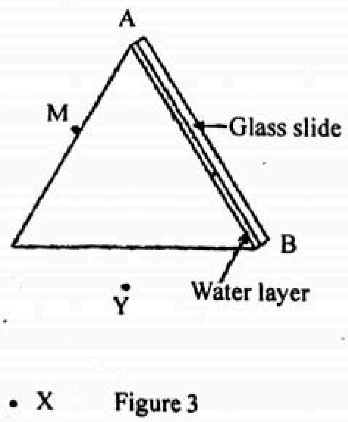

Figure 3

(i) Where would be the new location of the image of pin M relative to the image obtained in part (b) above.

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(ii) Draw the new emergent ray in figure 3 relative to X and Y and label it as $X'Y'$.

(d) Critical angles determined in part (b) and part (c) above are C_1 and C_2, respectively. Find an expression for the refractive index of water in terms of C_1 and C_2.

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04. A moving coil galvanometer with coil resistance R_G produces full scale deflection when a current of I_0 is passed through it.

(a) Write down an expression for the voltage (V_0) appearing across the terminals of the galvanometer in terms of R_G and I_0 when it shows a full scale deflection.

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(b) When a voltage (V_1) which is less than V_0 is appeared across the galvanometer it produces a deflection θ. If θ_m is the full scale deflection of the galvanometer, write-down an expression for V_1 in terms of θ, θ_m and V_0.

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(c) This galvanometer is to be converted to a voltmeter giving full scale deflection for a voltage V_2 which is much larger than V_0. If you are provided with a resistor having the suitable value R_1 show by drawing a diagram how you would connect this resistor to the galvanometer.

(d) Write down an expression for R_1 in terms of V_2, I_0 and R_G.

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(e) If $R_G = 20\ \Omega$ and $I_0 + 10$ mA find the value of the resistance R_1 necessary to convert this galvanometer to a voltmeter which gives a full scale deflection for 1V.

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(f) Also calculate the values of resistances R_2 and R_3 that are necessary to convert this galvanometer to voltmeters which give full scale deflection for 10V and 50V respectively.

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(g) Using the resistance values calculated in (e) and (f) and the galvanometer mentioned above, draw a circuit diagram of a multi-range voltmeter which can be used to measure voltages in three different ranges of 0 - 1V, 0 - 10V and 0 - 50V. Use a 3-way switch to select ranges.

(h) If this voltmeter is used in the 0-10V range to measure a voltage of the order of 5V appearing across a 2000 Ω resistor, would you expect to obtain the actual value? Explain your answer.

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PART B - Essay

Answer four questions only $[g = 10\,N\,kg^{-1}]$

01.

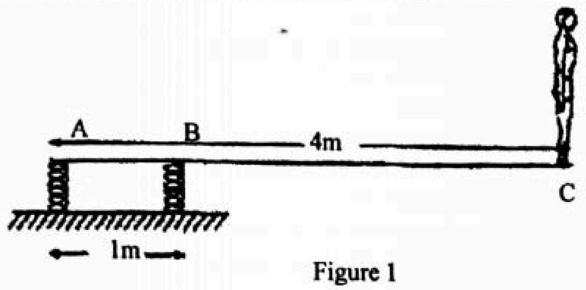


Figure 1

(a) A diver of mass 50kg is standing at the end (C) of a 4 m long horizontal diving board (AC) of negligible mass, mounted on two vertical springs, 1m apart, at A and B as shown in figure 1. Find the magnitude and direction of the forces acting on the board at A and B by springs.

(b) The diver performs a dive. Consider the motion of the centre of gravity (G) of the diver. Its path in indicated by a dotted line as shown in figure 2. The point G which is 4 m above the water surface at the beginning of the dive, enters the water surface at Y after completing the path in 2s. $XY = 2$ m. (Neglect air resistance.)

(i) Find the horizontal and vertical components of the initial velocity of G.
(ii) Calculate the maximum height reached by G from the water surface.
(iii) Calculate the following at the highest point of the path of the diver.
 (1) The translational kinetic energy.
 (2) The gravitational potential energy relative to the water surface.

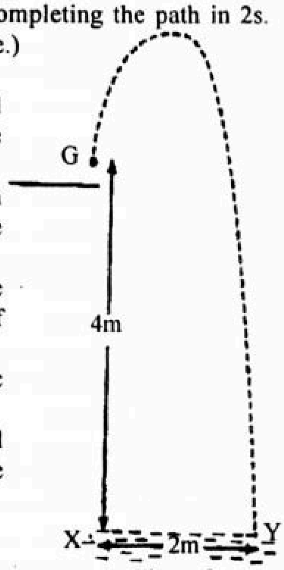
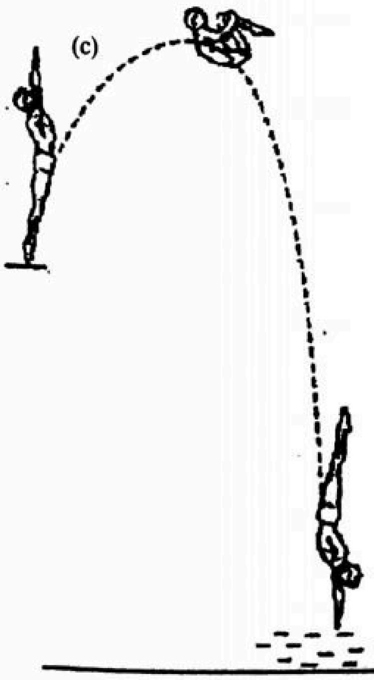


Figure 2

(c)



Diver also performs rotational motion about an axis (take as OP into the paper) passing through G. He controls his rotational motion by bending/extending his body to change the moment of inertia of the body. During the first 0.25s and last 0.75s of the motion the diver maintains his body in fully extended Position and during the rest of the time period of 1 s he maintains his body in tucked position. See figure 3.
(Take $\pi = 3.0$) The diver rotates around OP at a rate of 0.5 revolutions per second during first 0.25s.

(i) find the angular speed (ω_1) of the diver during first 0.25s.
If the diver rotates $2\frac{1}{2}$ revolutions around OP during the total time period of 2s find.

(ii) the angular speed (ω_2) when the diver is in fully tucked position.

(iii) The moment of inertia of the diver about OP in the fully tucked position. The moment of inertia of the diver about OP in fully extended position is 20 kg m^2.

(iv) the rotational kinetic energy of the body when the diver is in fully extended position.

02.

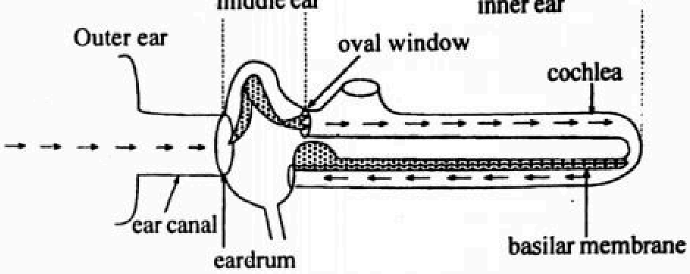


Figure 1

The ear converts the energy of sound waves into electrical energy. Therefore, the ear can be considered as a pressure transducer. The ear is divided into three parts, outer, middle and inner ear depending upon the role they play in response to incident sound. A crossectional view (simplified) of the ear is shown in figure 1.

The outer ear consists of an external auditory canal. It is open to atmosphere at one end and terminates at the eardrum at the other end. The auditory canal is 2.5 cm long, and the area of eardrum is 80mm^2. The auditory canal is equivalent to an organ pipe closed at one end. The ear is the most sensitive to sounds of frequencies around 3000Hz. The minimum intensity of sound which the ear can detect is 10^{-12}Wm^{-2}. Sound intensity level of 160dB may rupture the eardrum. The intensity (I) of a sound wave, expressed in terms of ist pressure amplitude (P_m), is given by $I = \frac{P_m^2}{2\upsilon\rho}$ where is υ the speed of sound in air and ρ is the density of air.

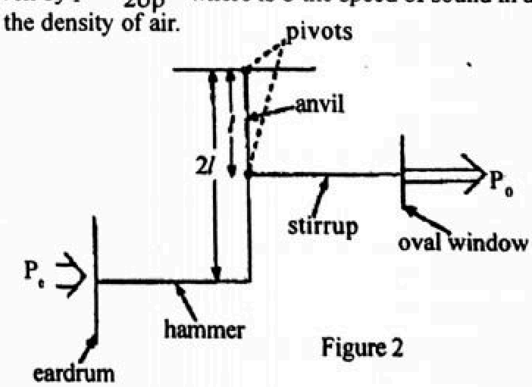


Figure 2

The important parts of middle ear are three small linked bones, called the hammer, the anvil and the stirrup, because of their respective shapes. These three bones function as a lever

system. Its one arm, hammer, is coupled to the eardrum. Its other arm, stirrup, is coupled to the oval window (area 4 mm^2) of the inner ear. A schematic representation of the lever and piston action of the middle ear is shown in figure 2.

The inner ear consists of a small spiral shaped tube called cochlea, filled with a fluid. In the figure 1, the cochlea is shown in 'straightend' form.

The cochlea is divided lengthwise into three canals which are separated from each other by membranes. As the pressure wave passes along the first canal it causes transverse displacements of the basilar membrane which separates the second canal from the third canal. It has been found that the basilar membrane is made of thousands of parallel fibres which run across it. The fibres of the basilar membrane towards the base of the cochlea are short and stiff. They vibrate very rapidly and are sensitive to high notes. In contrast, the fibres of the basilar membrane towards the apex of the cochlea are long and more flexible. Therefore they vibrate more slowly and are sensitive to low notes. This is how the inner ear resolves frequencies.

(a) What is the reason for treating the ear as a pressure transducer?

(b) (i) Around what sound frequency does the ear most sensitive?

(ii) Considering the auditory canal to be an organ pipe closed at one end, calculate its fundamental resonant frequency (speed of sound in air is 330m s^{-1})
Hence justify your answer given is (b) (i).

(iii) When the auditory canal resonates, is the pressure variation of the standing wave at the eardrum maximum or minimum? Give the reason for your answer.

(c) (i) Consider sound waves with intensity 10^{-12} W m^{-2}. Determine the corresponding pressure amplitude of the sound waves. (density of air is 1.25kg m^{-3} ; Take $\sqrt{33} = 5.5$)

(ii) Using the answer obtained in (c) (i) above, determine the force (F_e) acting on the eardrum.

(iii) Considering the lever action of the three bones, determine the force (F_0) generated on the oval window. (Use the data given in fig. 2 for this calculation.)

(iv) Hence calculate the pressure amplitude (P_0) on the oval window. Determine the factor by which the pressure is amplified.

(d) (i) How much sound intensity level might rupture the eardrum?

(ii) What intensity of sound does this correspond to?

(e) 'Higher frequencies stimulate the base region while lower frequencies stimulate the apex region of the basilar membrane.' Considering the fibres in the basilar membrane as uniform strings under tension justify the above statement.

03. Poiseuille's equation can be written as $Q = \frac{\pi \Delta \rho . r^4}{8 \eta l}$
Identify each physical quantity in the above equation.

(a) Show that the above equation is dimensionally correct.

(b) Crude oil of viscosity 0.9 kg $m^{-1}s^{-1}$ and density 9.0 x 10^2kg m^{-3} has to be delivered from a harbour to a refinery, using a straight horizontal metal tube of internal radius 20cm and length 1 km, at an average speed of 1.0 ms^{-1}.

(i) Calculate the pressure difference that should be maintained across the tube.

(ii) What is the minimum power needed to deliver the oil through the pipe at the given rate? (Take $\pi = 3.0$)

(iii) At what radial distances the speed of oil in the tube has its maximum and minimum values? What is the value of the minimum speed?

(c) The internal radius of the metal tube is decreased by 10% due to deposits. What percentage the pressure difference across the tube should be increased in order to deliver oil at the same rate mentioned in (*b*) above? (Take $\frac{10}{9} = 1.11$)

(d) Two smaller tubes having similar radii and lengths are now fitted to the end of the metal tube mentioned in (*b*) above, and oil is delivered to two other refineries, instead of the refinery mentioned in (*b*). If the length of a smaller tube is also 1km, and the pressure differences across all the tubes are equal, find the radius of a smaller tube.

04. (a) If the mass and the radius of the earth are M and R respectively, write down an expression for the gravitational potential at a point P, which is at a distance h (h > R) from the centre of the earth, in terms of M, h and the universal gravitational constant G. Assume that the gravitational potential is zero at an infinite distance from the centre of the earth.

(b) Suppose a small object of mass m is projected vertically upward from the point P with speed υ_i.

(i) Write down an expression for the total mechanical energy of the object at its starting point?

(ii) Obtain an expression for the maximum height H the object travels from the centre of the earth, in terms of h, G, M and υ_i.

(iii) Find an expression for the escape velocity υ'_e of the object in this situation, in terms of G, M and h.

(c) If υ_0 is the speed required to keep the object in a circular orbit at a distance h from the centre of the earth, show that $\upsilon'_e = \sqrt{2}\upsilon_0$.

(d) If, $M = 6 \times 10^{24}$ kg and R = 6400km, calculate the escape velocity υ_e at the surface of the earth.
Take $G = 6 \times 10^{-11}$ m^3 $kg^{-1}s^{-2}$ and $\sqrt{2} = 1.4$.

(e) The mean temperature of the surface of the earth is 280K. Calculate the root mean square speeds (υ_{rms}) for H_2 and O_2 molecules at this temperature.
Boltzmann constant = k = 1.4 x 10^{-23} J K^{-1}
Mass of a H_2 molecule = $^mH_2$ = 3 x 10^{-27} kg
Mass of a O_2 molecule = $^mO_2$ = 16 x $^mH_2$

(f) For a given temperature gas molecules have a range of speeds from very fast to very slow. To retain a given gas in atmosphere the condition, $6\upsilon_{rms} < \upsilon_e$, must be satisfied for that particular gas. Using the results obtained in (*e*) above, explain why oxygen gas exists in the earth's atmosphere but not hydrogen gas.

05. **Answer either part (A) or part (B) only.**

(A) Figure 1 shows a circuit diagram of a Wheatstone bridge.

V_0 is the voltage supplied to the bridge and a galvanometer can be connected across *AB* if needed.

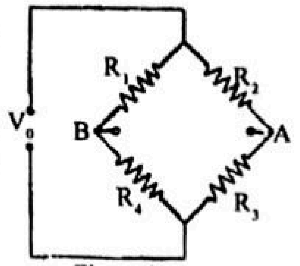


Figure 1

(a) Show that when the bridge is balanced $= \dfrac{R_1}{R_4} = \dfrac{R_2}{R_3}$

(b) Suppose $R_1 = R_2 = R_3 = R_4 = R$. The bridge is now made unbalanced by introducing a small resistance r into the R_3 arm so that $R_3 = R + r$. Show that under this condition a voltage of $\dfrac{Vr}{4R + 2r}$ will appear across *AB* (Note that when R >>> *r* the above expression reduces to $\dfrac{Vr}{4R}$).

(c) The resistance of the R_2 arm is now reduced to R - *r* while keeping R_3 arm at R + *r*. Show that by doing this change, the voltage across *AB* in (*b*) above can be doubled. (Assume R>>>*r*)

(d) Such increases or decreases of resistance occur, for example, when metal strips are subjected to elongations or contractions by the applications of external forces. If the volume and the resistivity of a metal strip do not change when elongated, show that its resistance increases.

(e) An accelerometer is constructed to measure accelerations of objects by fastening vertically an insulating rectangular rod *XY* to the upper-inner surface of a box and attaching a mass *M* firmly to the other end as shown in figure 2.

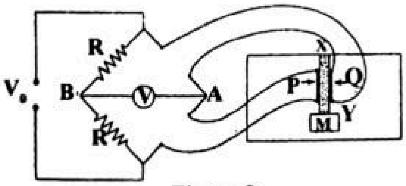


Figure 2

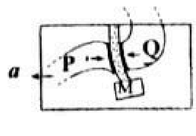


Figure 3

Two metal strips *P* and *Q* of resistance *R* are also fixed to either side of the rod. The ends of the strips are connected to two arms of a Wheatstone bridge as shown. When the box is placed on an accelerating object the rod and the strips bend as shown is figure 3.

(i) When the rod bends due to acceleration what would happen to the lengths of the strips *P* and *Q*.

(ii) If $V_0 = 5V$ and the magnitude of the fractional changes in resistance of the strips are same, and is equal to $\dfrac{1}{100}$, find the voltage generated across a voltmeter connected between *A* and *B*.

(iii) How would you calibrate such an accelerometer?

(B) (a) Show by drawing a circuit diagram how you would construct a NAND gate using a NOT gate and an 2-input AND gate.

(b) Considering voltages, prove that the circuit shown in figure 1 operates as an AND gate. (Assume that the voltage across a forward bias diode is 0.7 V.)

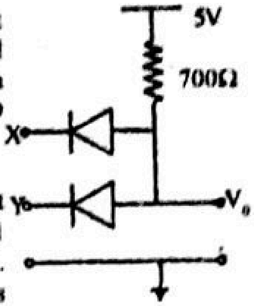


Figure 1

(c) Figure 2 shows a circuit diagram of a transistor circuit whose input B can be connected either to 5V or 0V. (Assume that when forward biased V_{BE} of the transistor is 0.7V.)

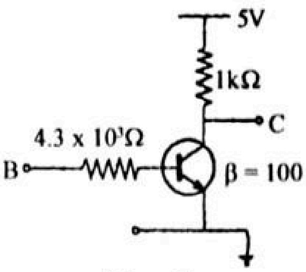


Figure 2

(i) If the current gain (β) of the transistor is 100 show that it is operating in the saturation mode when the input voltage is 5 V.

(ii) Considering voltages, prove that it operates as a NOT gate.

(d) The circuit in figure 3 is constructed by connecting the two circuits given in figure 1 and 2 together.

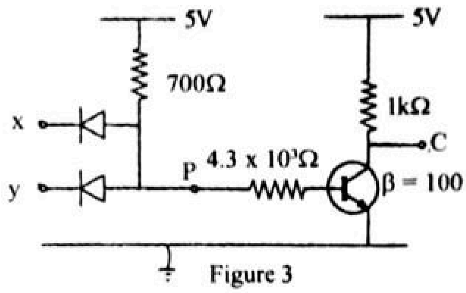


Figure 3

(i) Considering the logic levels at *P*, show that the circuit shown in figure 3 operates as a NAND gate (Note : This type of Diode Transistor Logic (DTL) gates are no longer in use and they are now replaced with TTL gates.)

(ii) When X = Y = 5V, what is the current through the base-emitter junction of the transistor?

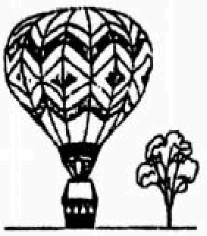
(e) Logic gates can be used to construct a circuit to operate an electric lamp in a room in the following manner. In this case the lamp is to be operated from two switches; switch *A* at the front door and switch *B* at the back door to the room. The lamp is to be lit when the switch A is ON and the switch is OFF or when the switch *A* is OFF and the switch *B* is ON. The lamps is to be OFF it both switches are OFF or ON. Using the same symbols *A* and *B* to represent corresponding logic variables of the switches,

(i) write down a logic expression for the output (*F*) of the circuit which satisfies the above requirements.

(ii) draw a logic circuit using gates which performs the above function.

06. **Answer either part (A) or part (B) only.**

(A) Figure shows a hot-air balloon inflated with ambient air at atmospheric pressure of 10^5 Pa and temperature of 27 °C. The inner volume of the balloon is 830 m^3. Treat air to be an ideal gas in all your calculations.



(a) (i) Determine the mass (m_1) of air inside the balloon at the above temperature. Hence calculate the density of air at 27 °C. (Take gas constant R = 8.3JK^{-1} mol^{-1}, the molar mass of air is 30g mol^{-1})

(Take $\frac{1}{83} = 0.012$)

(ii) To lift- off the air inside the balloon should be heated. Modern hot-air balloons heat the air by burning propane. The propane is stored in compressed liquid form, in light weight cylinders positioned in the balloon's basket. If the temperature of air inside the balloon is raised to T K, write down an expression for the mass (m_2) of air remaining inside the balloon at this temperature in terms of T. The pressure of the heated air remains the same as that of the atmospheric pressure.

(b) Calculate the upthrust acting on the balloon due to air (27 °C) outside. Neglect the volumes of the material of the balloon and all other contents including occupants.

(c) (i) If the total mass of the balloon excluding that of the hot air inside is 246 kg, determine the value of temperature T to which the air inside the balloon should be raised for the balloon to just lift-off the ground.
Hence determine the value of m_2.

(ii) Assuming that the heat released by burning propane is absorbed only by the air inside the balloon during the lift-off, estimate the heat supplied in this process. Take the mean temperature of air leaving the balloon to be

$\frac{300 + T}{2}$ K (The specific heat capacity C_p of air at constant pressure is 10^3 Jkg^{-1} K^{-1}).

(iii) If the amount of heat liberated when 1 kg of propane is completely burned is 87.5 MJ kg^{-1}, determine the mass of propane consumed in this process.

(B) One of the methods of measuring blood volume in human body is based on the measurement of the activity of a radioactive element added to the blood. In this method, a blood sample of known volume is removed from the body and a predetermined amount of radioactive element is added to it. This sample is then injected back into the human body. After a certain period, which is sufficient for the uniform distribution of radioactive material in the blood volume, a second blood sample is drawn and the radioactivity is measured. Blood volume can be calculated from the observed reduction of the activity.

A commonly used radioactive element in this procedure is ^{51}Cr, which has following properties.

Atomic number = 24; Half life = 28 d (28 days);
Specific activity (i.e. activity per unit mass) = 3.5 x 10^{15} Bq g^{-1}; Molar mass = 51 g mol^{-1}.

You are also given the following constant and equations.

Avogadro Number = 6 x 10^{23} mol^{-1} ; $T_{\frac{1}{2}} = \frac{0.7}{\lambda}$; $A(t) = \lambda N(t)$

where $T_{\frac{1}{2}}$ = Half life ; λ = Decay constant ; N(t) = Number of radioactive nuclei present at time t A(t) = Activity at time t

(a) Write down the number of protons and number of neutrons in a ^{51}Cr nucleus.

(b) Find the value of decay constant of ^{51}Cr in units of d^{-1} (per day)

(c) In a test to determine the blood volume of a patient of mass 70kg, a blood sample of 10 m ℓ was drawn from the patient and ^{51}Cr was added to it. If the activity inside the patient must be limited to the value 6.0 x 10^4 Bq per kg of body mass after injecting the ^{51}Cr added blood sample, calculate the maximum possible mass of ^{51}Cr that can be added to 10m blood sample.

(d) A mass of 1.53 x 10^{-10} g of ^{51}Cr was added to the 10 m blood sample. Calculate the number of ^{51}Cr nuclei added to the sample and the activity of the sample in Bq.
(Take 1 day = 9 x 10^4 s for your calculations.)

(e) 10 m blood sample as described in part (d) is injected back to the patient. After a sufficient time has elapsed another 10m sample was drawn from the patient and the measured activity of that sample was found to be 1000Bq. Assuming the ^{51}Cr added to the blood sample is uniformly distributed in the patient's blood volume and neglecting the number of decayed ^{51}Cr nuclei during this time period, calculate the volume of blood in the patient's body.

(f) If the decay of ^{51}Cr is **not neglected,** will the calculated blood volume in (*e*) be slightly greater than or less than the actual value of blood volume? Explain your answer.

(g) Find the time required for the activity of ^{51}Cr inside the patinet to become $\frac{1}{64}$ of the initial value.

(h) Explain why a radioactive element with a half-life of 10 s is not suitable for this procedure.