2.1.3. Expected answers and the scheme of marking for question paper I

Scheme of Marking for Paper I

Question No.	Answer	Question No.	Answer
01.	1	26.	2
02.	3	27.	1
03.	2	28.	3
04.	2	29.	5
05.	2	30.	3
06.	4	31.	1
07.	1	32.	3
08.	5	33.	4
09.	5	34.	All
10.	4	35.	4
11.	1	36.	1
12.	3	37.	2
13.	5	38.	4
14.	5	39.	3
15.	5	40.	5
16.	3	41.	3
17.	2	42.	3
18.	2	43.	1
19.	1	44.	1
20.	4	45.	5
21.	3	46.	4
22.	4	47.	2
23.	2	48.	5
24.	5	49.	3
25.	4	50.	5

Each correct answer carries 02 marks, amounting the total to 100.

2.2.2 Expected answers, scheme of marking, observations on the responses, conclusions and suggestions related to question paper II

★ **Observations regarding answering the paper II are based on graphs 2, 3, 4.1, 4.2 and 4.3.**

Part A - Structured Essay

1. When certain objects are packed in containers they do not occupy the entire volume of the container. This occurs due to the shape of the objects, and under such situations a fraction of the container volume is always empty and filled with air. Consider a container in the form of a cubical box of side length $8r$, which is fully packed with identical solid spheres of **radius** r in a regular manner as shown in figure (1). This is called **regular packing**.

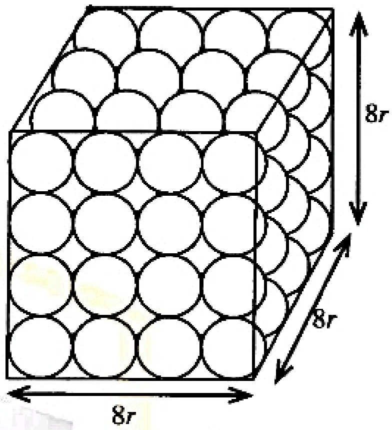


Figure (1)

(*a*) Find the number of spheres packed in the container.

64 .... **(01)**

(*b*) Obtain an expression for the total material volume of all spheres packed in the container in terms of r and π.

$\left(\frac{4}{3}\pi r^3\right) \times 64$ **OR** $\frac{256}{3}\pi r^3$ .... **(01)**

(*c*) When the container is completely filled with spheres, the ratio,

$\frac{\text{Total material volume of the spheres in the container}}{\text{Volume of the fully packed container}}$, is called the **packing fraction** (f_p) of the spheres and the volume of the fully packed container is called the **packed volume**. Find the packing fraction, f_p, for the above regular packing.

$$f_p = \frac{\frac{256}{3}\pi r^3}{512 r^3}$$

$$= \frac{\pi}{6}$$ .... **(01)**

(*d*) If m is the total mass of the spheres in the container, derive an expression for the ratio:

$\frac{\text{Total mass of the spheres}}{\text{Volume of the fully packed container}}$, in terms of m and r.

This ratio is called the **bulk density** (d_B) of the spheres.

$$d_B = \frac{m}{512 r^3}$$ .... **(01)**

(No marks for writing 8^3 instead of 512) (*e*) Write down an expression for the density (d_M) of the material of the spheres in terms of m, r and π.

$$d_M = \frac{m}{\frac{256}{3}\pi r^3} = \frac{3m}{256\,\pi r^3} \quad \text{.......... (01)}$$

(*f*) A student has decided to find the parameters f_p, d_B and d_M for green gram using an experimental method. In this case green gram is packed in a random manner and it is called **random packing**. See figure (2). The definitions mentioned in part (*c*), (*d*) and (*e*) for f_p, d_B and d_M are valid for random packing of items of any shape too.

First he inserted dry green gram into a measuring cylinder and obtained a packed volume of 50 cm³ of green gram as shown in figure (2).

Then he measured the mass of the packed volume 50 cm³ sample of green gram and it was found to be 3.8×10^{-2} kg.

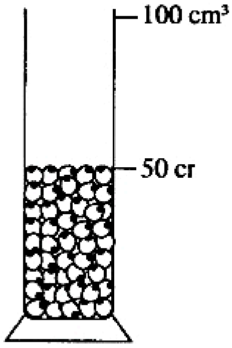


Figure (2)

Subsequently he introduced the green gram sample into a measuring cylinder containing 50 cm³ of water and found that the water level raised to 82 cm³ mark. See figure (3).

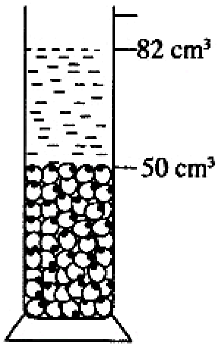


Figure (3)

(i) What is the material volume of green gram?

Material volume of green gram $= 32\ \text{cm}^3 = 3.2 \times 10^{-5}\ \text{m}^3$ .... **(01)**

(ii) Calculate the packing fraction (f_p) of green gram.

Packing fraction of green gram $f_p = \dfrac{32}{50}$

or $= 0.64$ .... **(01)**

(iii) Calculate the bulk density (d_B) of green gram in kg m⁻³.

Bulk density of green gram $d_B = \dfrac{3.8 \times 10^{-2}}{50 \times 10^{-6}}\ \text{kg m}^{-3}$

$= 7.6 \times 10^{2}\ \text{kg m}^{-3}$ .... **(01)**

(iv) Calculate the density (d_M) of material of green gram in kg m⁻³.

Material density of green gram $d_M = \dfrac{38 \times 10^{-3}}{3.2 \times 10^{-5}}\ \text{kg m}^{-3}$

$= 1.187 \times 10^{3}\ \text{kg m}^{-3}$

$(1.18 \times 10^{3} - 1.19 \times 10^{3})$ .... **(01)**

(*g*) A polythene bag is to be designed to pack 1 kg of green gram. Calculate the minimum volume of the bag needed.

Minimum Volume of the bag $\dfrac{1}{d_B}$ OR $\dfrac{50}{38} \times 1000\ \text{cm}^3 = 1315\ \text{cm}^3 = 1.315 \times 10^{-3}\ \text{m}^3$

$(1.31 \times 10^{-3}\ \text{m}^3 - 1.32 \times 10^{-3}\ \text{m}^3)$ .... **(01)**

2. You are asked to determine the dew point of air in the laboratory experimentally, and determine its relative humidity.

(*a*) Write down an expression for the relative humidity (RH) in terms of saturated vapour pressures.

$$RH = \frac{\text{Saturated (water) vapour pressure at dew point}}{\text{Saturated (water) vapour pressure at room temperature}} \times 100$$ .... **(01)**

(*b*) In addition to a polished calorimeter with a lid and a stirrer, what other items would you require to carry out this experiment?

Thermomrter (0 – 50 C°), Water, (beaker of) ice pieces,

[Glass plate, two stands, piece of blotting paper]

(All three underlined items correct) .... **(01)**

(*c*) Write down **two** factors that need to pay attention **before starting** the experiment in order to obtain a final result with better accuracy, and state experimental precautions that you would take to minimize them.

	Factors	Experimental Precautions
(1)	Exhale air altering the moisture level around the calorimeter	Keeping a **glass plate to block the exhaled air** OR wearing a mask ....**(01)**
(2)	Fans, winds, air conditioners disturbing the dew formation on the surface of the calorimeter	Switch off fans, close windows and air conditioners .... **(01)**

(*d*) Small pieces of ice are used for this experiment. Give reasons for this.

Lowering or raising of the temperature of water can be done slowly or in a controlled manner, **OR** Formation or dissappearance of dew can be observed well, **OR** Dew point can be measured accurately **OR** Dew point can be noted accurately, **OR** Temperature at which dew appears can be recorded accurately .... **(01)**

(*e*) What practical difficulties would you face if several pieces of ice are added to water at a time?

It is not possible to observe the disappearance of dew as a thin layer of water is formed on the surface of the calorimeter .... **(01)**

(*f*) Exactly at what instants do you take the readings in this experiment?

At the instants when dew just begins to apper and disapper, **OR** when the shine begins to disappear and appear. .... **(01)**

(*g*) **What is the reason for using the calorimeter with a lid in this experiment?**

It will prevent spilling of cold and saturated air present inside the calorimeter and interfering with the dew formation. .... **(01)**

(*h*) **What is the other reading that you should take in this experiment?**

Room temperature .... **(01)**

(*i*) **When the temperature of a certain laboratory was 28 °C, its dew point was found to be 24 °C. Using the following table, determine the relative humidity of the laboratory.**

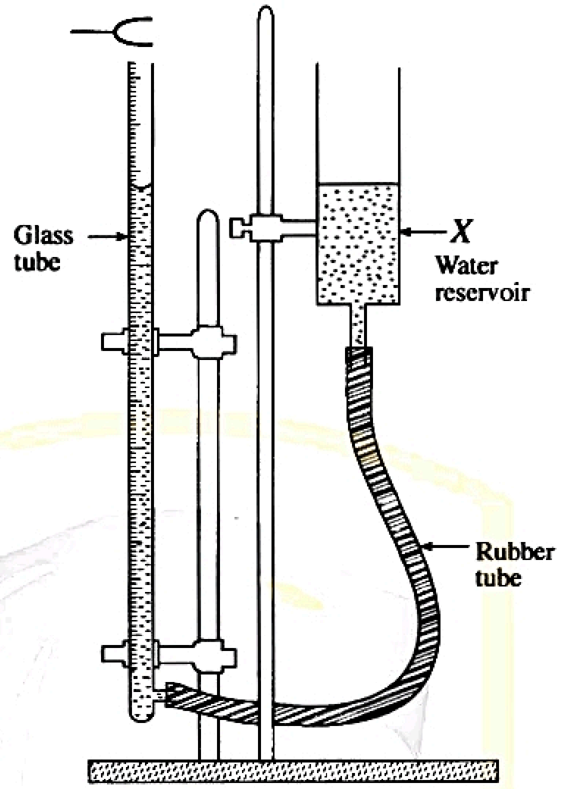
Temperature (°C)	20	22	24	26	28	30	32
Saturated water vapour pressure (mmHg)	17.53	19.83	22.38	25.20	28.35	31.82	35.66

$$\text{Relative humidity} = \frac{22.38}{28.35} \times 100$$

$$= 79\%$$

(78.9% – 79%) .... **(01)**

3. Figure shows an alternative apparatus to find the speed of sound in air using a resonance tube with one end closed. The principle of this apparatus is similar to the principle of the apparatus normally used in the school laboratory. The resonance tube in this apparatus is a glass tube with a calibrated scale. The water level in the resonance tube is raised and lowered by raising and lowering of a water reservoir X which is connected to the resonance tube with a flexible rubber tube.



(*a*) What type of wave is formed inside the tube at resonance?

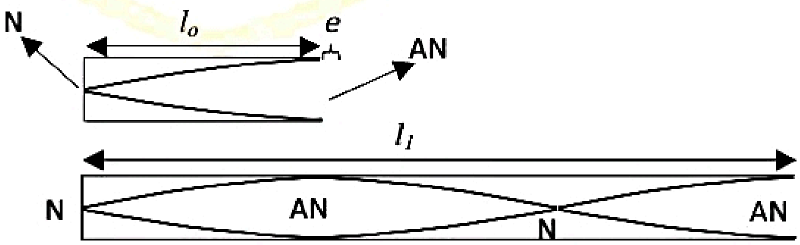
Standing wave **OR** Stationary wave .... **(01)**

(*b*) You are given a tuning fork of known frequency f and asked to obtain the resonant lengths l_0 and l_1 corresponding to the fundamental note and the first overtone respectively.

(i) Draw the wave patterns for the two modes of vibrations, and mark the lengths l_0, l_1, end correction e, Nodes (N) and Anti-nodes (AN).
(You are expected to draw the tube for the first overtone).

Fundamental note:

First overtone:



To draw both wave patterns (look for the length of overtone approximately three times) .... **(01)**

If all the labelling are correct (at least in one diagram)
(Instead of AN, A can also be accepted) .... **(01)**

(ii) (1) If λ is the wavelength corresponding to the fundamental note, write down an expression for λ in terms of, l_0 and e.

$$\lambda = 4(l_0 + e)$$ .... **(01)**

(2) Write down a similar expression for the wave length corresponding to the first overtone

$$\lambda = \frac{4}{3}(l_1 + e)$$ .... **(01)**

(3) If v is the speed of sound in air, derive an expression for v in terms of the known and measured quantities.

$$l_1 - l_0 = \frac{\lambda}{2}, \quad \Rightarrow \quad v = f\lambda$$

$$v = 2f(l_1 - l_0)$$ .... **(01)**

(*c*) Before taking the measurement for l_0, the water level in the resonance tube has to be raised upto the top. Explain the reason for this.

To detect the fundamental note <u>without missing</u> it. **OR** to get the <u>fundamental note first</u>.

(*d*) Write down **two** major differences in the experimental procedure when using the apparatus given in the question compared to the method adapted when using apparatus generally available in school laboratory.

(1) Tube is fixed (or water level movable) .... **(01)**

(2) Measuring scale is fixed (or calibrated tube) **OR** does not need a metre ruler.

(Both correct) .... **(01)**

(*e*) At room temperature (28 °C), when a 512 Hz tuning fork is used, the corresponding lengths of the resonance for fundamental note and the first overtone are found to be 15.5 cm and 50.5 cm respectively.

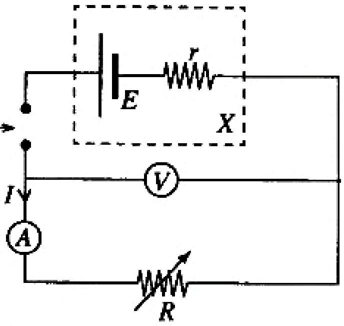
Calculate the speed of sound in air at room temperature.

$$v = 2 \times 512(50.5 - 15.5) \times 10^{-2} \text{ m s}^{-1} \quad \Rightarrow \quad v = 358.4 \text{ m s}^{-1}$$

Correct substitution .... **(01)**

Final answer .... **(01)**

4. The given circuit can be used in a school laboratory to experimentally determine the e.m.f. (E), and the internal resistance (r) of a dry cell X using a graphical method.



The experimental procedure consists of measuring the potential difference V across the terminals of the cell for different values of I using a voltmeter with very high internal resistance.

(*a*) Write down an expression for V in terms of I, E, and r.

$V = E - Ir$ .... **(01)**

(*b*) (i) Name the variable resistor that is available in the school laboratory, which could be used for this experiment.

Rheostat

(No marks for resistance box) .... **(01)**

(ii) The key S should be used properly to obtain expected results from this experiment.

(1) What is the type of key that is most suitable to be used as S?

Tap key **OR** drawing a diagram to show a tap key .... **(01)**

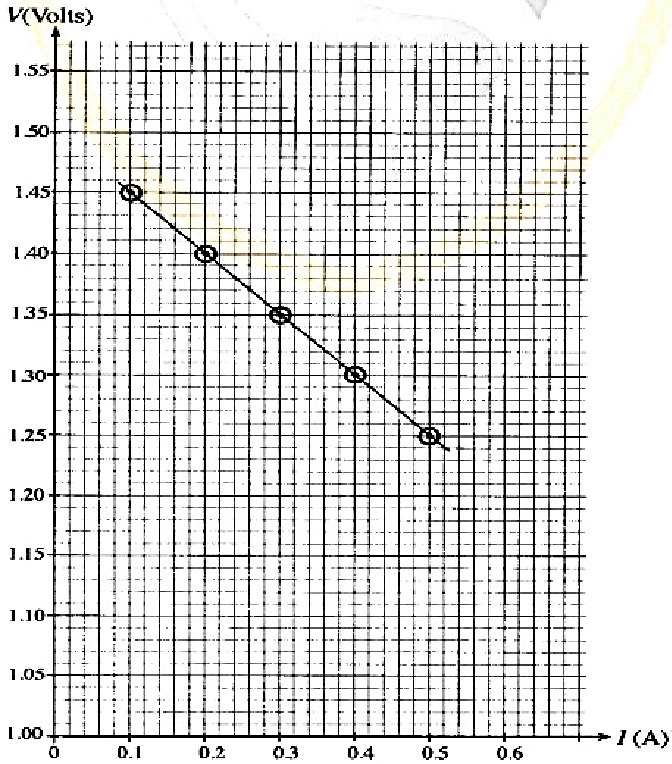
(2) What is the experimental procedure you adapt when operating the key?

Vary R while keeping S open and **close the key momentarily** when observing or taking the I and V readings. .... **(01)**

(iii) How do you confirm experimentally that the cell has not run down while carrying out the experiment?

After taking the final reading go back to the first reading and check its value again to see if it is different. .... **(01)**

(*c*) A graph of V against I plotted using a set of data obtained from such an experiment is shown below.



(i) Use the graph to find the following.

(1) Internal resistance, r of the cell.

$$\text{Gradient of the graph} = \frac{1.24 - 1.44}{0.52 - 0.12}$$

$$= (-)\ 0.5\ \Omega \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \textbf{(01)}$$

(Select two points which are far apart)

(2) E.m.f. E of the cell.

Intercept $= E = 1.5$ V .... **(01)**

When giving the mark look for the extension in the graph to determine the intercepts **OR** using one point from the graph and substituting it into the equation and finding E.

(ii) Use the values obtained under (*c*)(i) and the expression obtained under (*a*) to deduce the current (I_{SC}) through the cell if it is short circuited.

Using $V = E - IR$ and taking V as zero when the cell is short circuited

$E = I_{sc}\, r$ **OR**

$$I_{SC} = \frac{1.5}{0.5} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \textbf{(01)}$$

$$= 3.0\ A$$

(*d*) A supply voltage in the range, 8.6 V – 9.0 V will have to be applied to operate a certain electronic item properly. The resistance across the supply voltage terminals of the electronic item is 30 Ω. Suppose you have a choice of selecting a single dry cell battery having E = 9 V and r = 10 Ω or a combination of six dry cell batteries each having E = 1.5 V and r = 0.2 Ω and connected in series, for the operation of the above electronic item. Using the data given in this part, explain how you would select a proper battery.

When the dry cell battery having E = 9V and r = 10 Ω is connected, the terminal voltage (V) across the electronic device is given by $V = \left(\frac{9}{30 + 10}\right) \times 30 = 6.75$ V

When six 1.5 V dry cell batteries having E = 9V and r = 0.2 × 6Ω are connected, the terminal voltage (V) across the electronic device is given by $V = \frac{9}{3.0 + 1.2} \times 30$

= 8.65 V, Therefore, only six 1.5V dry cell batteries can provide a voltage greater than 8.5 V.

Correct substitution for any of the voltage calculation .... **(01)**

If both calculated voltage values are correct and correct argument .... **(01)**

Alternative method :

Instead of calculating the potential differences across the device one can argue in terms of current

Conversion of voltage range (8.6 V – 9.0 V) to corresponding current range (0.287 A – 0.30 A) .... **(01)**

Calculation of current from each dry cell and for correct argument .... **(01)**

Part B - Essay

5. A particle of mass m_1 is fixed to the rim of a horizontal ring of radius r and negligible mass as shown in figure (1). POQ is a vertical axis passing through the centre O of the ring.

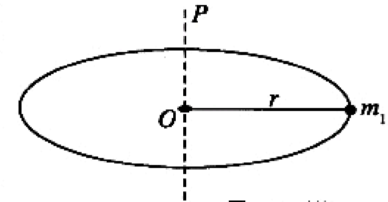


Figure (1)

(*a*) (i) Write down an expression for the moment of inertia I_1 of the particle about the vertical axis POQ in terms of m_1 and r.

(ii) Another particle of mass m_2 is now fixed to the rim of the ring which is diametrically opposite to m_1, and the system is rotated about the axis POQ with a constant angular speed ω. If I_2 is the moment of inertia of mass m_2 about the axis POQ, write down an expression for the total rotational kinetic energy (E) of the system.

(iii) If I_0 represents the total moment of inertia of the above system in (*a*) (ii) about the axis POQ, using the expression obtained in (*a*)(ii) show that $I_0 = I_1 + I_2$.

(*b*) Instead of m_1 and m_2, 10 identical particles, each of mass m, are now fixed to the rim of the ring with equal spacing. If I is the moment of inertia of a particle about the vertical axis POQ, write down an expression for total moment of inertia (I_T) of the system about the vertical axis POQ.

(*c*) Now, the ring described in (*b*) above is fixed onto an axle of negligible moment of inertia and coinciding with the vertical axis POQ using symmetrically fixed spokes of negligible mass as shown in the figure (2). The system is then started rotating from rest at time $t = 0$ in a horizontal plane about the axis POQ with a constant angular acceleration α and reached a constant angular speed ω.

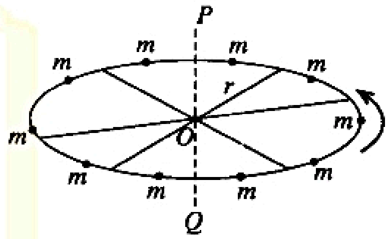


Figure (2)

(i) (1) Obtain an expression for the time t taken by the system to reach the constant angular speed ω.

(2) How many revolutions have been made by the system when it reaches the constant angular speed ω?

(ii) Write down an expression for the centripetal force (F) acting on one particle when it is rotating about the axis POQ with a constant angular speed ω.

(*d*) The structure of the merry-go-round shown in figure (3) which is at rest is similar to the structure of the system described in (c) above. However, instead of fixed masses m, the system has 10 chairs occupied by riders and hung by chains of negligible mass. The moment of inertia of the merry-go-round, **without riders and chairs**, about the axis POQ is 32 000 kg m^2.

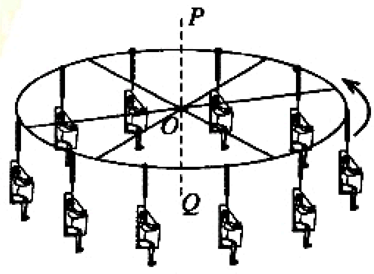


Figure (3)

Consider a situation where the merry-go-round is rotating about the axis POQ with a constant angular speed of 12 revolutions per minute with all the chairs being occupied by riders. When the merry-go-round rotates, all the chains are inclined to the vertical by an angle θ, and figure (4) shows the situation with respect to one rider.

Use $\pi = 3$ for relevant calculations.

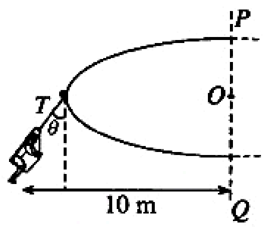


Figure (4)

(i) If the riders are of mass 70 kg each and the chairs are of 20 kg each, calculate the total moment of inertia of the system about the axis POQ. When calculating the moment of inertia assume that the total mass of the rider and his chair **is concentrated** at a horizontal distance of 10 m from the axis POQ.

(ii) Calculate the value of θ.

(iii) What is the total rotational kinetic energy of the system?

(*a*) (i) Moment of inertia of the particle about the vertical axis *POQ*,

$I_1 = m_1 r^2$ .... (01)

(ii) Total rotational kinetic energy of the system,

$E = \frac{1}{2} I_1 \omega^2 + \frac{1}{2} I_2 \omega^2$ **OR**

$E = \frac{1}{2} m_1 r^2 \omega^2 + \frac{1}{2} m_2 r^2 \omega^2$ .... (01)

(iii) $\frac{1}{2} I_0 \omega^2 = \frac{1}{2} I_1 \omega^2 + \frac{1}{2} I_2 \omega^2$ .... (01)

$\therefore I_0 = I_1 + I_2$

(*b*) (i) Total moment of inertia of the system about the vertical axis *POQ*.

$I_T = I_1 + I_2 + ... + I_{10}$

$= mr^2 + mr^2 + ...$

$= 10\, mr^2 = 10I$ .... (01)

(*c*) (i) (1) For a system rotating with constant angular acceleration α, the relation between initial and final angular speeds is

$\omega_0 = \omega_0 + \alpha t$

$\omega = 0 + \alpha t$

$\therefore \omega = 0 + \alpha t$

$\therefore t = \frac{\omega}{\alpha}$ .... (01)

(2) Total angle θ through which the system rotated is given by,

$\omega^2 = \omega_0^2 + 2\alpha\theta$ **OR** $\theta = \omega_0 t + \frac{1}{2}\alpha\alpha t^2$ .... (01)

$\theta = \frac{\omega^2}{2\alpha}$

Number of revolutions made by the system when it reaches angular speed $\omega = \frac{\theta}{2\pi}$

$= \frac{\omega^2}{4\pi\alpha}$ .... (01)

(ii) Centripetal force acting on one particle $F = m\omega^2 r$ .... (01)

(*d*) (i) The moment of inertia of the system about the axis *POQ*

$= 32{,}000 + (70 + 20) \times 10^2 \times 10$ (01)

$= 122{,}000$ kg m^2 (01)

(ii) Take the mass of a chair with the passenger as m, then,

$T\cos\theta = mg$ .... **(01)**

$T\sin\theta = ma$
$= m\omega^2 r$ } (Either of the above equation) .... **(01)**

$$\therefore \tan\theta = \frac{\omega^2 r}{g}$$

$$= \left(\frac{12 \times 2\pi}{60}\right)^2 \times \frac{10}{10}$$ (Correct substitution) .... **(01)**

$$= 1.44$$

$\theta = 55°\ \ (55° - 13')$ .... **(01)**

(if $\pi = 3.14$ then $\tan\theta = 1.58$ and $\theta = 57°\ (57° - 57°\ 40')$)

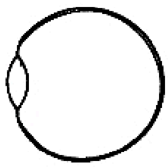
(iii) Total rotational kinetic energy of the system $= \frac{1}{2} I\omega^2$

$= \frac{1}{2} \times 122{,}000 \times 1.44$

$=$ 87840 J (87840 J – 87850 J)......... **(01)**

(if $\pi = 3.14$ then 96220 J (96220 J – 96230 J))

6. The effective focal length of the cornea and the eye-lens can be considered as the focal length of an eye. The muscles controlling the curvature of the lens permit the eye to focus on the retina light from objects at different distances from the eye. The figure shows a simplified diagram of the eye with an eye-lens of effective focal length. When the eye muscles are relaxed the focal length of a healthy eye of a child is about 2.5 cm. The near point of his eye is at a distance of 25 cm.



(Copy the diagram given in the figure and use it when drawing ray diagrams.)

(*a*) Draw a ray diagram for the situation where light from a far away object is focused onto the retina of the eye of the child with healthy eye when his eye muscles are relaxed. What is the distance between the eye-lens and the retina?

(*b*) Draw a ray diagram for a situation where a point source of light is placed at the near point, is clearly seen by the child with healthy eye. Calculate the focal length of the eye at this instant.

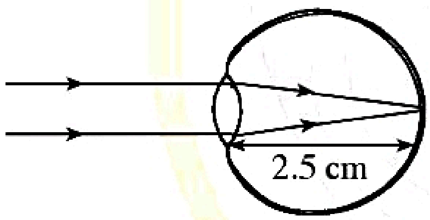
(*c*) Another child has the focal length equal to that of the healthy child when the eye muscles are relaxed and also has the focal length calculated for the situation in (*b*). However, the position of his retina is located 0.2 cm behind the position of the retina of the healthy child.

(i) Using the image produced by a point source of light as mentioned in (b) above, indicate his near point and far point by drawing two separate ray diagrams. Calculate the distances from the eye-lens to the near point and to the far point of this child.

(ii) Sketch a ray diagram illustrating as to how the required correction can be done using a suitable lens. Calculate the focal length of the corrective lens needed.

(*d*) When a person becomes older the ability to change the focal length of eyes gets weaker and the distance to the near point of the eye increases. If the child mentioned in part (*c*) above would face such a situation, what is the type of additional corrective lens that the child should wear (convergent/divergent)? Give reasons for your answer.

(*a*)

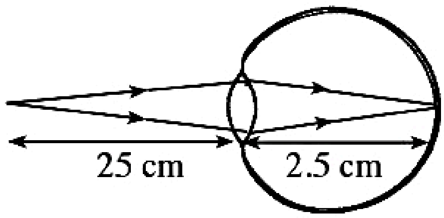


Drawing the correct ray diagram .... **(01)**

(Look for **two parallel lines with arrows** to the point image at the retina)

(The distance between the eye lens and the retina = 2.5 cm .... **(01)**

(*b*)



Drawing the correct ray diagram .... **(01)**

(Look for **two lines with arrows** from a point source to the point image at the retina)

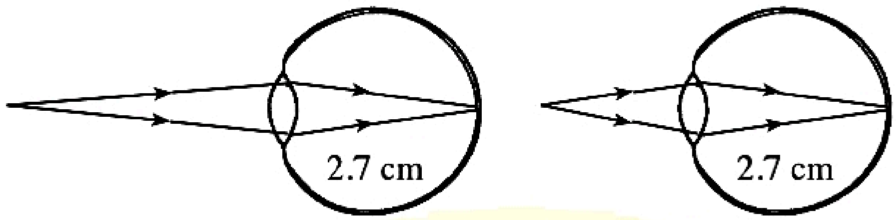
Let *f* be the focal length, then

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad (u = 25\text{ cm} ; v = -2.5\text{ cm})$$

$$\frac{1}{-2.5} - \frac{1}{25} = \frac{1}{f}$$ (Correct substitution) .... (01)

$f = -2.273$ cm OR 2.273 cm .... (01)
(2.27 cm – 2.30 cm)

(*c*) (i)



(a) Far point (b) Near point

Drawing the correct ray diagram for an object at the **far point** .... (01)

(Look for two lines with arrows from a point source to the point image at the retina)

Drawing the correct ray diagram for an object at the **near point** .... (01)

(Look for two lines with arrows from a point source to the point image at the retina)

Calculation of the distance to the far point : $f = -2.5$ cm, $v = -2.7$ cm, $u = ?$

$$\frac{1}{-2.7} - \frac{1}{u} = \frac{1}{-2.5}$$ (Correct substitution) .... (01)

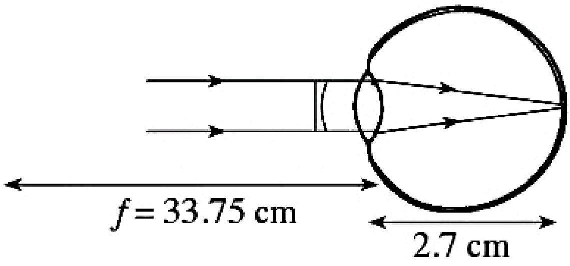
$u = 33.75$ cm .... (01)

Calculation of the distance to near point : $f = -2.273$ cm, $v = -2.7$ cm, $u = ?$

$$\frac{1}{-2.7} - \frac{1}{u} = \frac{1}{-2.273}$$ (Correct substitution) .... (01)

$u = 14.373$ cm (14.25 cm – 14.40 cm). .... (01)

(ii) Ray diagram for the required correction with a corrective lens



Choosing a divergent lens .... **(01)**
Correct ray diagram with diverging lens .... **(01)**
(Look for two dotted lines from the point source and parallel lines with arrows to the point image at the retina)

$f = 33.75$ cm .... **(01)**

OR

Focal length of the corrective lens ; $u = -2.5$ cm, $v = -2.7$ cm, $f = ?$

$$\frac{1}{-2.7} - \frac{1}{-2.5} = \frac{1}{f} \quad \textbf{OR} \quad \frac{1}{33.75} - \frac{1}{\infty} = \frac{1}{f}$$ .... **(01)**

$f = 33.75$ cm

(*d*) Additional corrective lens should be a converging lens

Reason :

The image formed by the eye lens should be moved forward to coincide with the retina **OR** When the eye lens gets weaker the image of an object at the normal near point will be formed behind the retina. Therefore, light passing through the lens has to be converged to the retina.
.... **(01)**

7. Write down Poiseuille's equation for the rate of flow, Q, of a liquid through a horizontal cylindrical narrow tube under a pressure difference of ΔP. Identify all the other symbols you used.

Under the above condition, the resistance exerted by the tube against the rate of flow of the liquid, Q, can be defined as the flow resistance $R = \frac{\Delta P}{Q}$.

(*a*) What physical quantities associated with the tube and the liquid determine the flow resistance R?

(*b*) When a liquid flows under pressure differences of ΔP_1, ΔP_2 and ΔP_3, through three horizontal narrow tubes connected in series as shown in figure (1), the flow resistances exerted by the tubes are R_1, R_2 and R_3 respectively. Using the definition given above for R show that the flow resistance, R_0, of the system can be written as $R_0 = R_1 + R_2 + R_3$. (Neglect edge effects.)

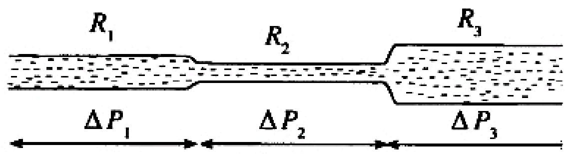


Figure (1)

(*c*) When a liquid flows through two horizontal narrow tubes connected in parallel under a common pressure difference ΔP as shown in figure (2), the flow resistances exerted by the tubes are R_1 and R_2. Show that the flow resistance R_0 of the system can be written as $\frac{1}{R_0} = \frac{1}{R_1} + \frac{1}{R_2}$. (Neglect end effects.)

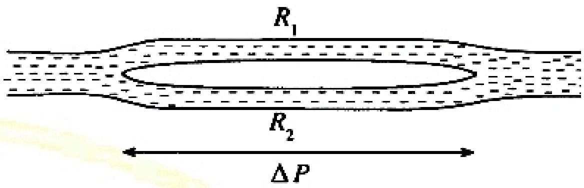


Figure (2)

(*d*) Figure (3) shows a set of horizontal narrow tubes A, B, C, D and E connected between the point X and a common reservoir Y so that a liquid can flow from X to Y. The pressures at X and Y are maintained at constant values. The flow resistance of each tube is labelled in the diagram in units of mmHg s/cm^3. Tube B is divided into two tubes C and D of equal flow resistances. This simplified model may also be used to illustrate the blood flow through arteries and veins.

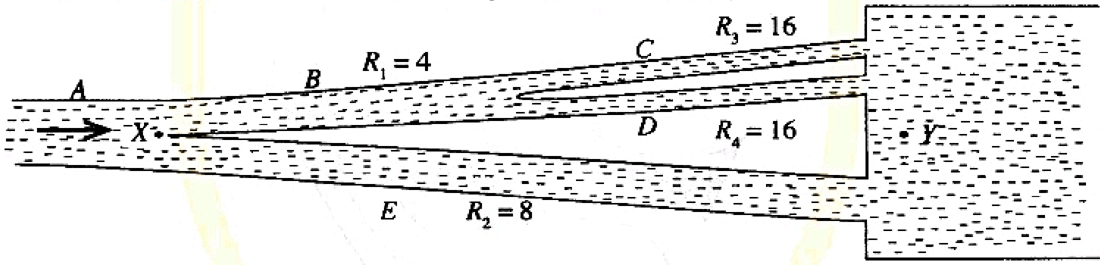


Figure (3)

Give the answers to parts (i), (ii) and (iii) in terms of the given units. (Take $\pi = 3$)

(i) (1) Calculate the flow resistance, due to the system of tubes B, C and D between the points X and Y.

(2) Calculate the flow resistance, due to the system of tubes, B, C, D and E between the points X and Y.

(ii) If the flow rate of the liquid across X is 6 cm^3/s, calculate the pressure difference between X and Y.

(iii) Using the above results, find the flow rate of the liquid through tube E.

(iv) If the length of tube E is 2 cm find the internal radius of tube E. The viscosity of the liquid is 4.0×10^{-3} Pa s [Take 1 mmHg = 133 Pa]

(*e*) If the temperature of one of the tubes in the system given in part (*d*) is reduced, explain what would happen to the flow rate of the liquid in that tube. Neglect the changes in radius and length of the tube.

Poiseuille's equation : $Q = \dfrac{\pi \Delta P r^4}{8\eta l}$ .... (01)

η – Viscosity of the liquid $\quad l$ – Length of the tube

r – Radius of the tube

All three correct .... (01)

[Flow resistance against the flow Q, $R = \dfrac{\Delta P}{Q} = \dfrac{8\eta l}{\pi r^4}$]

(*a*) Flow resistance is determined by the :

Coefficient of viscosity of the liquid

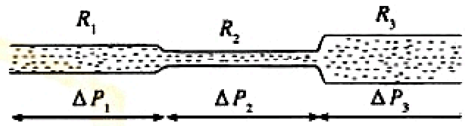
Length of the tube

Radius of the tube.

All three correct .... (01)

(*b*) $\Delta P = \Delta P_1 + \Delta P_2 + \Delta P_3$ .... (*a*)

$R_0 Q = R_1 Q + R_2 Q + R_3 Q$ .... (*b*)



OR

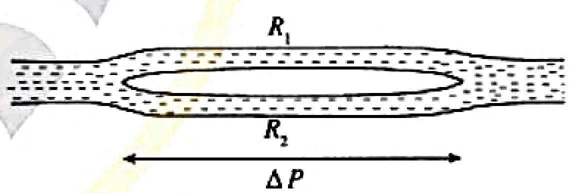
$$\left[\frac{\Delta P}{Q} = \frac{\Delta P_1}{Q} + \frac{\Delta P_2}{Q} + \frac{\Delta P_3}{Q} \text{ .... } (b)\right]$$

$R = R_1 + R_2 + R_3$

For equation (a) .... (01)

For equation (b) .... (01)

(*c*) ΔP is common for both tubes



$Q = Q_1 + Q_2$

$\dfrac{\Delta P}{R_0} = \dfrac{\Delta P}{R_1} + \dfrac{\Delta P}{R_2}$ .... (01)

$\dfrac{1}{R_0} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$

(*d*)(i) (1) $R_{CD} = \dfrac{1}{\dfrac{1}{16} + \dfrac{1}{16}}$ **OR** 8 mmHg s/ cm^3 (Correct substitution or answer) .... (01)

$R_{BCD} = 8 + 4$

12 mmHg s/ cm^3 .... (01)

(2) Flow resistance R due to system of tubes B, C, D and E

$R = \dfrac{1}{\dfrac{1}{12} + \dfrac{1}{8}}$ = 4.8 mmHg s/ cm^3 (Correct substitution).............................. (01)

(ii) Pressure difference between X and Y

$\frac{\Delta P}{Q} = R$ **OR** $\frac{\Delta P}{6} = 4.8$ .... **(01)**

(iii) Flow rate through E

$$Q = \frac{\Delta P}{R} = \frac{28.8}{8}$$

$= 3.6\ cm^3/s$ .... **(01)**

(iv) Radius of the tube E

$$Q = \frac{\pi \Delta P r^4}{8\eta l}$$

$3.6 \times 10^{-6} = \frac{3 \times 28.8 \times 133 \times r^4}{8 \times 4.0 \times 10^{-3} \times 2 \times 10^{-2}}$ (Correct substitution) .... **(01)**

$r = 6.69 \times 10^{-4} m = 0.669\ mm$ .... **(01)**

$(6.68 \times 10^{-4} m - 6.70 \times 10^{-4} m)$

(If $\pi = 3.14$ $r = 6.619 \times 10^{-4} m$

$(6.61 \times 10^{-4} m - 6.62 \times 10^{-4} m)$

(*e*) If the temperature inside a tube is reduced the viscosity will increase and hence the flow rate will decrease .... **(01)**

8. Read the following passage and answer the questions.

Induction heating technology is of choice in many industrial, domestic and medical applications due to its advantages such as less heating time, localized heating, direct heating and efficient energy consumption. The operating principle of induction heating is based on the law of the electromagnetic induction discovered by Michael Faraday in 1831. The two major components in an induction heating system are a coil of wire (often a copper coil) producing a time varying magnetic field upon receiving a high frequency alternating current, and an electrically conducting material that generates heat. The magnetic field also changes its direction as the direction of the alternating current changes. When a conducting material is exposed to such a time-varying magnetic field, current loops called eddy currents are induced in the conducting material. As the magnetic field changes its direction rapidly the eddy currents also change their directions rapidly. The eddy currents always form closed loops inside conducting materials in planes perpendicular to the varying magnetic field. Eddy currents, generate Joule heat (I^2R type heat) due to the existence of resistance of the material.

When the magnetic field created is stronger or when the electrical conductivity is higher or when the rate of change of magnetic field is larger, the eddy currents that are developed become larger. The eddy currents which are generated by high frequency alternating current in the coil will exist only within a limited thickness near the surface of the material due to what is called skin effect.

The skin effect is the tendency of any high frequency electric current to distribute itself in a conductor with the current density being largest near the surface of the conductor and decreasing very rapidly with the depth of the conductor. This thickness across which eddy currents are distributed becomes even smaller due the mutual attraction between the alternating current in the coil and the eddy current loops. This is called the proximity effect. In addition to the Joule heating, an additional heat is also produced within the material due to a phenomenon called hysteresis effect. It occurs only in ferromagnetic materials such as some stainless steel, cast iron, nickel, etc. In response to the varying magnetic field produced by the alternating current, the magnetic domains in these materials repeatedly change their orientations. The energy required to turn them around finally is converted to heat. The rate at which the heat is generated due to hysteresis effect increases with the frequency of the varying magnetic field. Commercially available induction heating systems operate at frequencies approximately from 60 Hz to about 1 MHz and deliver power in the range from a few watts to several Megawatts.

The cookers that are available in the market as induction cookers operate on this principle. In an induction cooker, a coil of copper wire is mounted just under the surface of the cooker top where the cooking pot is placed, without touching it, and an alternating electric current is sent through the coil. The entire bottom of the cooking pot itself acts as the conducting material that generates the heat. The varying magnetic field produced by the coil enters the bottom of the cooking pot creating eddy currents and hysteresis losses, generating heat. In order to make use of both effects for heat generation, the cooking pots or the bottoms of the cooking pots are made of ferromagnetic materials such as some stainless steel or cast iron.

(*a*) State Faraday's Law of electromagnetic induction in words.

(*b*) Name **two** fields of application where induction heating is used.

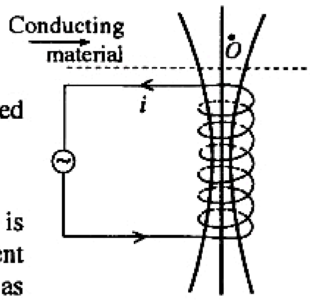
(*c*) Write down the **two** heating processes involved in the induction heating.

(*d*) Write down **three** factors which give rise to larger eddy currents.

(*e*) Write down the **two** effects which limit the eddy currents to be within a limited thickness near the surface of the material.

(*f*) Copy the given diagram and answer the following questions.

The direction of the alternating current in a coil at a certain instant of time is shown in the figure. Consider a situation where the magnitude of this current is increasing with time. A conducting material is placed just above the coil as shown in the figure.



(i) Show the direction of the magnetic field created in this situation by drawing an arrow on one field line.

(ii) Draw one loop of eddy current in the material near the position O and show the direction of the eddy current when the alternating current is increasing.

(iii) Use Lenz's law to explain how you determined the direction of the eddy current loop that you have drawn in (ii) above.

(*g*) Explain how the increase of the frequency of alternating current, increases the rate of heating in the material.

(*h*) Consider a situation where a time-varying magnetic field enters a disk of radius R, thickness b and resistivity ρ. If the flux density B of the applied magnetic field varies sinusoidally as $B = B_0 \sin \omega t$ where B_0 is the amplitude of the flux density of the magnetic field, ω is the angular frequency and t is the time, then based on a very simplified model the average power P generated by the eddy currents in the disk can be given by $P = k\, B_0^2\, \omega^2$ where $k = \frac{\pi R^4 b}{16\rho}$. If $k = 0.5\ \text{m}^4\ \Omega^{-1}$, $\omega = 6000\ \text{rad s}^{-1}$ and $B_0 = 7.5 \times 10^{-3}\ T$, calculate the average power generated in the disk.

(*i*) In transformers, the core is heated up due to eddy currents and it contributes to energy loss in the form of heat. How is this energy loss minimised in transformers?

(*a*) **Faraday's law** : The emf induced in a circuit is directly proportional to the time rate of change of magnetic flux through the circuit.

OR

When the magnetic flux connecting a circuit changes, an electromotive force is induced in the circuit proportional to the rate of change of the flux. .... **(01)**

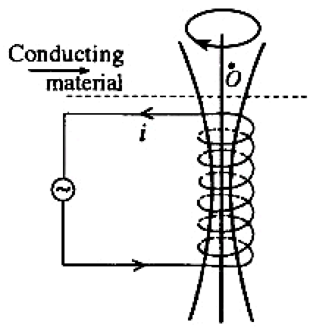
(*b*) Industrial, domestic and medical applications (Any two correct) .... **(01)**

(*c*) Joule heating (I^2R type), Hysteresis effect (Change of orientation of magnetic domains) (Both correct) .... **(01)**

(*d*) Magnetic field created is stronger, electrical conductivity is higher, the rate of change of magnetic field is larger. (All three correct) .... **(01)**

(*e*) Skin effect, Proximity effect (Both correct) .... **(01)**

(*f*) (i)



Drawing a correct arrow on a fields line as shown .... **(01)**

(ii) Drawing an eddy current loop as shown .... **(01)**
Showing the direction of the eddy current with an arrow. .... **(01)**

(iii) According to the Lenz's law, the induced current and induced emf in a conductor are in such a direction as to set up a magnetic field that opposes the change in the magnetic field that produced them. .... **(01)**

The magnetic field produced by the coil is increasing in the upward direction therefore to oppose this magnetic field the induced eddy current should be in the opposite direction of the current in the coil. .... **(01)**

(*g*) The increase of the frequency of alternating current increase the rate of change of magnetic flux in the conducting material. .... **(01)**

Increase in the rate of change of magnetic flux increases the magnitude of eddy current in the material. .... **(01)**

(*h*) $P = kB_0^2\omega^2 = 0.5 \times (7.5 \times 10^{-3})^2 \times (6000)^2$ W $= 1012.5$ W **OR** $P = 1013$ W

Correct substitution .... **(01)**
Correct answer .... **(01)**

(*i*) The <u>core (conducting parts) is often laminated</u> (that is, they are built up in thin layers separated by a non – conducting material such as lacquer or a metal oxide) .... **(01)**

9. Answer either part (A) or part (B) only.

(A) (*a*) Write down an expression for the energy (W) dissipated in a resistor of resistance R when a current of magnitude I is passed through it for a period of time t.

(*b*) An electrical fuse is a small element consisting of a thin metal wire. Electrical fuses are connected in series with electrical/electronic circuits to avoid damages caused to them due to the passage of currents larger than the recommended current for the circuits (due to over-load currents and short circuits). When the current through the fuse in a certain circuit becomes larger than the recommended current in the circuit, the fuse burns (melts) and disconnects the circuit from the power source. The electrical fuses are selected so that their ratings are equal to the recommended currents in the circuits.

(i) Figure (1) shows how a fuse is connected to a circuit of load resistance R. Current in a certain fuse is rated as 5 A. If the length of the fuse wire is 3 cm, and its radius is 0.1 mm (area of cross-section $\sim 3 \times 10^{-8}\,\text{m}^2$) and the resistivity of the material of the wire at 25 °C is $1.7 \times 10^{-8}\,\Omega$ m, calculate the resistance of the fuse wire at room temperature of 25 °C.

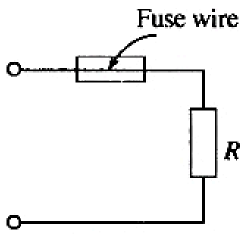


Figure (1)

(ii) When the fuse is operated at the rating mentioned in (i), at steady state, the entire heat generated by the fuse wire is dissipated to the surrounding without burning the fuse. Calculate the power dissipated by 5 A fuse in that manner. Take the average value of the resistance of the fuse wire over the temperature range is equal to five times the resistance calculated under (*b*)(i).

(iii) A test performed by manufacturers of electrical fuses is to determine the amplitude of a current pulse needed to melt (burn) the fuse wire approximately in one millisecond. Considering the rectangular current pulse of one millisecond duration shown in the figure (2), calculate the peak current I_0 of the pulse needed to melt the fuse wire given in (*b*)(i). Assume that the heat dissipation to the surroundings under this condition is negligible. Take the mass of the fuse wire given in (*b*)(i) as 7.5×10^{-6} kg, and the average value of resistance of the fuse wire as five times the resistance calculated under (*b*)(i). Specific heat capacity of the material of the fuse wire is 390 J kg^{-1} °C^{-1}. Melting point of the material of the fuse wire is 1 075 °C.

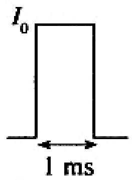


Figure (2)

(iv) Consider a situation in which a load circuit with an applied voltage of 230 V as shown in the figure (3) is short circuited at XY. Calculate the current through a 5 A fuse under this situation. Using the results obtained in (*b*)(iii), show that the fuse will melt before 1 millisecond. (Assume that the current produced is a rectangular current pulse.)

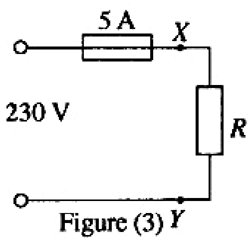


Figure (3)

(v) A rectangular narrow current pulse of 500 A occurring for a duration of 1 μs passes through a 5 A fuse. In this situation, will the fuse get burnt? Justify your answer using an appropriate calculation.

(*a*) $W = I^2Rt$ .... **(01)**

(*b*)(i) $R = \dfrac{\rho l}{A}$ .... **(01)**

$= \dfrac{1.7 \times 10^{-8} \times 3 \times 10^{-2}}{3 \times 10^{-8}}$ (Correct substitution) .... **(01)**

$= 1.7 \times 10^{-2}\,\Omega$ .... **(01)**

(ii) $P = I^2R$

$= 5^2 \times (1.7 \times 10^{-2}) \times 5$ .... **(01)**

$= 2.125\ \text{W}$ .... **(01)**

(iii) $I_0^2 Rt = mc\,\Delta\theta$ ($mc\,\Delta\theta$ (Symbols have their usual meaning)................ **(01)**

(For equating mc $\Delta\theta$ to electrical energy)

$I_0^2 = \dfrac{(7.5 \times 10^{-6}) \times 390 \times 1050}{(1.7 \times 10^{-2}) \times 5 \times 10^{-3}}$ (Correct substitution) .... **(01)**

$= 3.6132 \times 10^4$

$I_0 = 1.90 \times 10^2\text{A}$ ($1.900 \times 10^2\ \text{A} - 1.901 \times 10^2\ \text{A}$) .... **(01)**

(iv) Current through 5 A fuse $= \dfrac{230}{1.7 \times 10^{-2} \times 5}$ .... **(01)**

$= 2.706 \times 10^3\ \text{A}$

($2.70 \times 10^3\ \text{A} - 2.71 \times 10^3\ \text{A}$)

Since this current is greater than I_0 in part (iii), the fuse will melt before 1 millisecond .... **(01)**

(A ward this mark for the above statement only if two respective current values are correct)

Alternative method :

If t is the time required to melt the fuse, then

$I^2Rt = mc\,\Delta\theta$

$t = \dfrac{mc\,\Delta\theta}{I^2R}$

$t = \dfrac{(7.5 \times 10^{-6}) \times 390 \times 1050}{(2.706 \times 10^3)^2 \times 1.7 \times 10^{-2} \times 5}$ .... **(01)**

$= 4.93 \times 10^{-4}\ s$ .... **(01)**

$= (4.93 - 4.94)$

$\therefore$ The fuse will burn before one millisecond. .... **(01)**

(v) No

Justification :

Energy needed to melt the fuse wire $mc\,\Delta\theta = 7.5 \times 10^{-6} \times 390 \times 1050$ .... **(01)**

$= 3.07$ J

Energy dissipated in the fuse $= 500^2 \times (1.7 \times 10^{-2}) \times 5 \times 10^{-6}$ .... **(01)**

$= 2.125 \times 10^{-2}$ J

This value is very much less than the energy needed (3.07 J) to melt the wire. Therefore,

the fuse will not melt. .... **(01)**
(For comparison of the above two values)

Alternative method :

If $\Delta\theta$ is the increase in temperature of the fuse, $\Delta\theta = \dfrac{I^2 Rt}{ms}$

$$\Delta\theta = \frac{500^2\ (1.7 \times 10^{-2}) \times 5 \times 10^{-6}}{(7.5 \times 10^{-6}) \times 390}$$ .... **(01)**

$= 7.26$ °C

$\therefore$ Final temperature attained by the fuse wire is $(25+7.26)$°C $= 32.26$°C **(01)**

and it will not melt. .... **(01)**

(B) Figure (1) shows the circuit symbol of an operational amplifier having open loop voltage gain A.

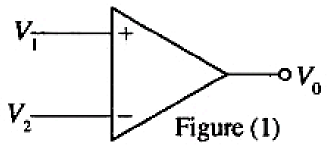


Figure (1)

(*a*) Write down the expression for the output voltage V_0 in terms of V_1, V_2, and A.

(*b*) If the positive and negative output saturation voltages of the operational amplifier are ± 15 V and $A = 10^5$, calculate the minimum input voltage difference which will drive its output into saturation.

(*c*) (i) Draw the output voltage waveform when the given triangular voltage signal of peak amplitude 5 V is applied to the + input of the circuit as shown in figure (2), and label its peak voltage values.

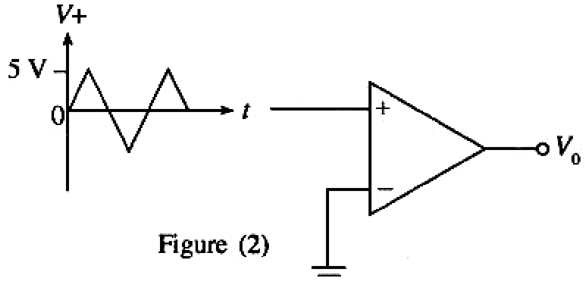


Figure (2)

(ii) The circuit in figure (2) is now modified as shown in figure (3). When S_1 is closed and S_2 is open the circuit will produce the output waveform shown in the figure (3) for the input triangular signal. Considering the actions of circuit elements in figure (3), explain the reasons for differences, if any, between the output voltage waveform shown in figure (3) and the waveform drawn by you in (*c*)(i). What is the peak voltage of the output in figure (3)?

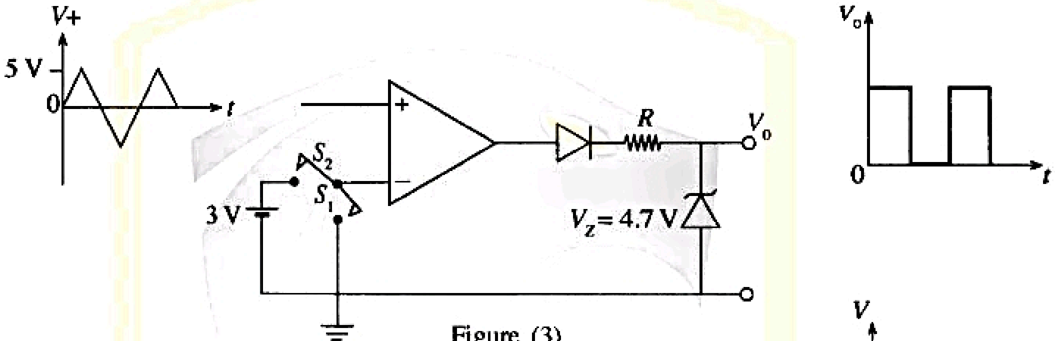


Figure (3)

(iii) Now a voltage of +3 V is applied to the − input of the operational amplifier in figure (3) by opening S_1 and closing S_2. When a hypothetical voltage waveform shown in figure (4) is applied to the + input of the operational amplifier, draw the output waveform expected from the circuit and label the magnitude of the output voltage.

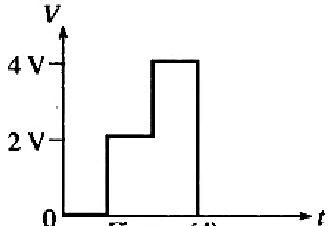


Figure (4)

(*d*) A certain blood cell counting system operates as follows. The blood is diluted by a known proportion in a proper type of solution, and allowed to flow through a small aperture X of the order of 50 μm diameter placed in between two electrodes S and T as shown in the figure (5). Blood cell counting is based on the fact that the electrical resistivity of blood cells is higher than the electrical resistivity of the solution.

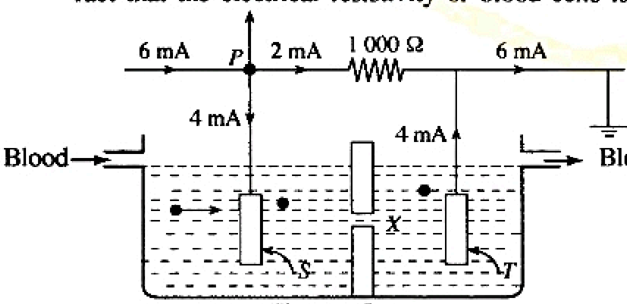


Figure (5)

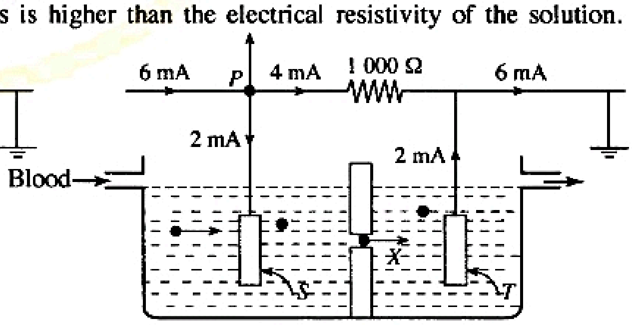


Figure (6)

A constant current of 6 mA is passed through the system as shown in figures (5) and (6). Currents through 1000 Ω resistor and the electrodes when the **solution** passes through the aperture X is shown in figure (5). Figure (6) shows the currents through 1 000 Ω resistor and the electrodes when a **blood cell** is going through the aperture X. The point P of the circuits shown in figures (5) and (6) is connected to + terminal of the operational amplifier in the circuit shown in figure (3) with S_1 open and S_2 closed. The output V_0 is connected to a pulse counter. (Not shown in the figure.)

(i) What are the voltages at point P in figures (5) and (6)?

(iii) Draw the output voltage waveform of the circuit in figure (3) relevant to (ii) above.

(iv) What does the counter output indicate if a diluted blood stream is allowed to flow through the aperture X?

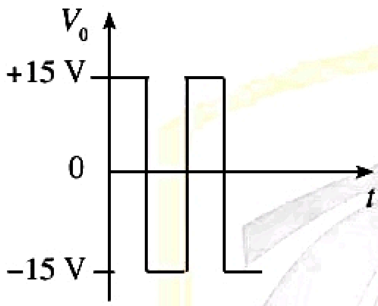
(*a*) $V_0 = A(V_1 - V_2)$ .... **(01)**

(*b*) $(V_1 - V_2)_{min} = \dfrac{+15}{10^5}$ .... **(01)**

$= 1.5 \times 10^{-4}\ V$ .... **(01)**

(**OR** correct value in any other appropriate voltage units)

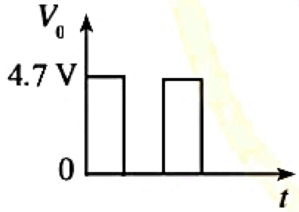
(*c*)(i)



A waveform symmetrical about *t* axis as shown .... **(01)**

Labeling of peak voltage values ±15 V as shown .... **(01)**

(ii)



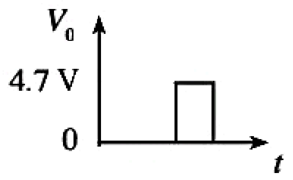
[Difference between two waveforms : (not asked)
(1) Out put waveform in c(i) has equal negative and positive half cycles but the waveform in c(ii) has only positive cycles.
(2) Peak voltage of the waveform in c(i) is (±)15V, but the peak voltage of the waveform in c(ii) is + 4.7 V]

Reasons :

(1) The diode is reversed biased during negative half cycles of the waveform, and does not allow the negative half cycle of the waveform to pass through. No current will pass through when it is in reversed biased .... **(01)**

(2) Zener diode will limit the peak voltage of the output waveform in c(ii) to 4.7V .... **(01)**

(iii) Output waveform



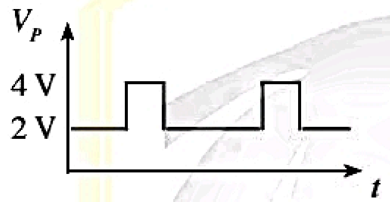
Output waveform as shown (a shape of pulse) .... **(01)**

Magnitude of the output voltage (4.7 V) .... **(01)**

(*d*) (i) Voltage at *P* in figure (5) = 2 V .... **(01)**

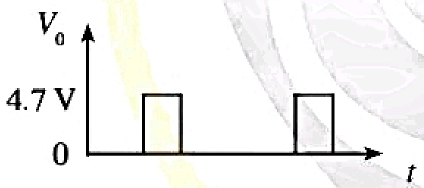
Voltage at *P* in figure (6) = 4V .... **(01)**

(ii)



Shape of the waveform (single pulse is enough) .... **(01)**

(iii)



Output voltage waveform relevant to (ii) above, as shown (Single pulse is enough) .... **(01)**

Labeling the peak voltage as above .... **(01)**

(iv) The counter output indicates the number of blood cells passed through the aperture .... **(01)**

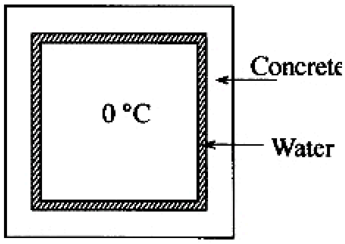
10. Answer either part (A) or part (B) only.

(A) (*a*) (i) Briefly explain how heat is absorbed when the physical state of a material is changed from solid state to the liquid state.

(ii) 10 mega Joules of excess thermal energy produced by a certain thermal power plant is to be stored as latent heat in an insulated **solid** block of zinc which is maintained at its melting point of 420 °C. If the entire excess energy is used to melt zinc, calculate the minimum mass of solid zinc necessary for this purpose.

Specific latent heat of fusion of zinc is 1.15×10^5 J kg^{-1}.

(*b*) The temperature inside a certain outdoor closed storage room in a cold country is to be maintained at 0 °C when the outside temperature is at –30 °C. The room is thermally insulated with 20 cm thick concrete walls. The inner surfaces of the walls are in contact with a uniform water layer of sufficient thickness maintained at 0 °C as shown in the figure. Water is stirred internally to avoid formation of static frozen ice layers. (Assume that the stirring process does not add any heat to water.)



(i) Explain briefly how the temperature of the room can be maintained in 0 °C upto sometime using this method.

(ii) Calculate the minimum mass of the water layer which will ensure that the 0 °C temperature is maintained in the room upto 10 hours and only 25% of the mass of water is converted to ice during this time period.

Total mean surface area of all the walls is 120 m^2.
Thermal conductivity of concrete = 0.8 W m^{-1} °C^{-1}. Specific latent heat of fusion of ice = 3.35×10^5 J kg^{-1}.

(iii) Suppose the above mentioned entire water layer is frozen due to some unforseen reason and a uniform ice layer of thickness 5 cm is formed on the inner surface of concrete walls. Calculate the rate at which the heat from the 0 °C room begins to flow out as soon as the ice layer is formed. Thermal conductivity of ice = 2.2 W m^{-1} °C^{-1}. For calculations, assume that the total mean surface area of the ice layer through which the heat flows out is also 120 m^2.

(*a*) (i) The latent heat is partly used to overcome the forces of attraction between the molecules. .... **(01)**

(ii) The minimum mass m required is given by,

$$m \times 1.15 \times 10^5 = 10 \times 10^6$$

$m = 86.95$ kg (86.95 – 86.96) .... **(01)**

(*b*) (i) Latent heat of water is capable of compensating the heat loss through concrete without changing the temperature of water or room. .... **(02)**

(02 Or Zero)

(ii) Amount of heat lost (Q) through concrete given by,

$Q = 0.8 \times 120 \times \dfrac{30}{20 \times 10^{-2}} (3600 \times 10)$ .... **(01)**

Use of equation $\dfrac{dQ}{dt} = kA\dfrac{d\theta}{dL}$ in the above expression .... **(01)**

$Q = 5.184 \times 10^8$ J

If the minimum mass needed is m

Heat to be given out by water $= m \times 120 \times \frac{25}{100} \times 3.35 \times 10^5$ .... **(01)**

(For multiplying the above expression by $\frac{25}{100}$)

$\therefore m \times \frac{25}{100} \times 3.35 \times 10^5 = 5.184 \times 10^8$ .... **(01)**

(for equating the two expression)

$m = 6.190 \times 10^3 \text{kg}$ .... **(01)**

$= (6.189 \times 10^3 - 6.191 \times 10^3)$

(iii) Let θ be the temperature at the ice-concrete interface. Then,

$$\frac{dQ}{dt} = k_1 A \frac{0 - \theta}{L_1}$$ .... **(01)**

$$= k_2 A \frac{\theta - (-)30}{L_2}$$

(For both equations)

$$\left(\frac{L_1}{k_1 A} + \frac{L_2}{k_2 A}\right) \frac{dQ}{dt} = 30$$ .... **(01)**

$$\left(\frac{5 \times 10^{-2}}{2.2 \times 120} + \frac{20 \times 10^{-2}}{0.8 \times 120}\right) \frac{dQ}{dt} = 30$$ .... **(01)**

$$\frac{dQ}{dt} = 1.320 \times 10^4 \text{ Js}^{-1}$$ .... **(01)**

$(1.319 \times 10^4 - 1.320 \times 10^4)$ .... **(01)**

(B) Radioisotope Thermoelectric Generators (RTGs) are used to generate electricity in space-crafts, satellites etc. An RTG consists of two subsystems.

(1) Thermal source:
It is a container of alpha particle emitting radioactive source. The kinetic energy produced by all the alpha particles is converted to thermal energy and absorbed by the container.

(2) Energy conversion system:
It is a thermoelectric generator which converts thermal energy absorbed by the container into electrical energy.

Consider an RTG of a certain space-craft which uses ^{238}Pu in the form of plutonium oxide (PuO_2) as the radioactive source. The radioactive source contains 2.38 kg of PuO_2 for which the fraction of ^{238}Pu in PuO_2 is 0.9 at the launch of the space-craft. The thermal energy absorbed per radioactive decay of ^{238}Pu by the container is 5.5 MeV. Half life of ^{238}Pu is 87.7 years and the corresponding decay constant is $0.0079\ y^{-1}$ ($= 2.5 \times 10^{-10}\ s^{-1}$). Avogadro number is 6.0×10^{23} atoms per mole.

(i) Find the initial activity in Bq of the radioisotope source at the launch of the space-craft.

(ii) If the efficiency of conversion of thermal power into electrical power is 7%, find the electrical power in the RTG at the launch of the space craft ($1\ MeV = 1.6 \times 10^{-13}\ J$).

(iii) Find the activity of the radioisotope source by the end of the 10 years mission of the space-craft. (Take $e^{-0.079} = 0.92$)

(iv) Find the electrical power produced by the RTG at the end of the mission.

(v) Find the percentage loss of the electrical power after the mission.

(vi) Give **one** advantage of using RTGs in space-crafts.

(i) The amount of ^{238}Pu in the source $= 2380 \times 0.9g$ .... **(01)**

No.of atoms in the source $No = \dfrac{2380 \times 0.9 \times 6.0 \times 10^{23}}{238}$ .... **(01)**

$N_o = 5.4 \times 10^{24}$ atoms

Initial activity $A_o = N_o\lambda$ .... **(01)**

$= 5.4 \times 10^{24} \times 2.5 \times 10^{-10} s^{-1}$ .... **(01)**

$= 1.35 \times 10^{15} Bq$ .... **(01)**

(ii) Let E = Energy absorbed by the container in one decay

Thermal power produced $= A_oE$ .... **(01)**

$A_oE = 1.35 \times 10^{15} \times 5.5 \times 1.6 \times 10^{-13}$ .... **(01)**

$= 1188\ W$

Electric power produced at the launch of the space craft

$= 1188 \times \dfrac{7}{100}$ .... **(01)**

$= 83.2\ W$ .... **(01)**

$= (83.1\ W - 83.2\ W)$

(iii) Activity of the source after 10 years of the mission (A),

$$A = A_o e^{-\lambda t}$$

$$= 1.35 \times 10^{15} \times e^{-0.0079 \times 10}$$

(Writing equation or substitution) .... **(01)**

$$= 1.35 \times 10^{15} \times 0.92$$

$$= 1.24 \times 10^{15} \text{ Bq}$$ .... **(01)**

(iv) Electric power produces by the RTG at end of the mission $= 1.24 \times 10^{15} \times (5.5 \times 1.6 \times 10^{-13}) \times \frac{7}{100}$

OR $83.2 \times \frac{A}{A_o} = \frac{83.2 \times 1.24 \times 10^{15}}{1.35 \times 10^{15}}$ .... **(01)**

$= 76.4$ W .... **(01)**

(76.3 – 76.5)

(v) Percentage loss of electric power after the mission $= \frac{83.2 - 76.4}{83.2} \times 100$

$= 8\%$ .... **(01)**

$=$ (8% – 8.2 %)

(vi) 1. RTG can be used when solar energy is not available

2. Can Get electric power for a longer period in compared with other electrical sources.

3. Can be used without maintenance

Any of the above .... **(01)**