Elasticity

1994 A/L

1) Distinguish between the elastic limit and the proportional limit of a given material.

Two uniform steel wires of equal lengths of 0.5 m and cross-sectional area of $0.5\ cm^2$ and $0.2\ cm^2$ respectively, are connected together to form a compound wire of length 1m.

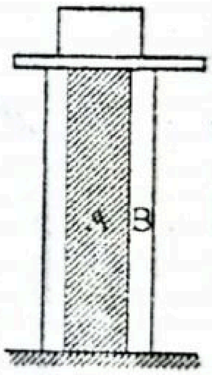
The Young's modulus and the proportional limit of steel are $2.0 \times 10^{11}\ Nm^{-2}$ and $2.5 \times 10^{8}\ Nm^{-2}$ respectively.

(i) What is the maximum mass that can be hunged by the compound wire so that the proportional limit is not exceeded? Calculate the total elongation of the compound wire in this situation.

(ii) If the two wires are joined at the ends so that they are parallel to each other and forms a compound wire of length 0.5 m, what is the maximum mass that can be hunged by the compound wire so that the proportional limit is not exceeded?

1995 A/L

2) A vertical support is made of two solid co-axial metal cylinders A and B each of length 5 m as shown in the figure. The inner cylinder A is of radius 10 cm and the outer cylinder B has an internal radius of 10 cm and an external radius of 15 cm. The lower end of the support is fixed rigidly to the horizontal floor and a horizontal plate of negligible mass is placed on the upper end. A weight of 2.2×10^6 N is kept on the plate and the plate remains horizontal.



The young's modulii of the materials of A and B are $1.0 \times 10^{11}\ Nm^{-2}$ and $1.2 \times 10^{11}\ Nm^{-2}$, respectively

(i) What is the ration of the forces acting on A and B?

(ii) What is the decrease in length of the support due to the weight that is placed on the plate?

(iii) Suppose at a time when the weight is not present on the plate, the temperature of the support has gone up by 20^0C. Calculate the increase in the lengths of A and B under this situation.

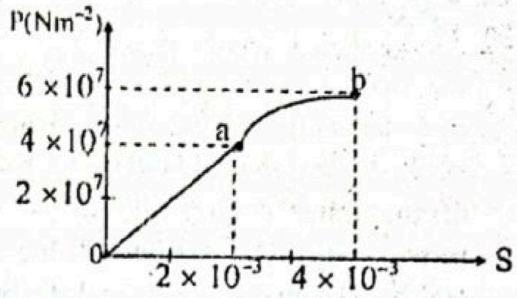
Linear expansivities of the materials of A and B are $2.0 \times 10^{-5}\ °C^{-1}$ and $1.0 \times 10^{-5}\ °C^{-1}$ respectively.

(iv) Now if the weight (2.2×10^6 N) is brought back on to the plate while keeping the temperature of the support at the value mentioned in (iii), show that the length of the support again becomes 5 m.

[If You expression in (iv) above contain terms such as $(5 + \Delta l)$ and if Δl is less than 0.005 m you may neglect Δl]

1997 A/L

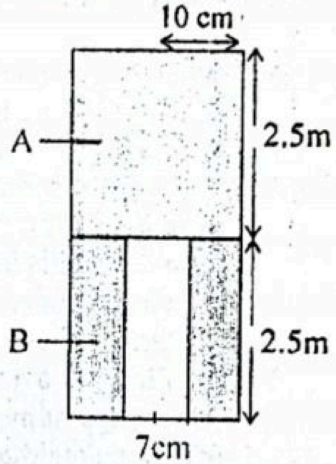
3) Figure shows the stress (P) – strain (S) curve obtained for a material for all possible stress values.



(i) Identify the points a and b on the curve.

(ii) Calculate the Young's modulus of the material. What is the energy stored per unit volume when the strain in the material is 2×10^{-3}?

(iii) A vertical cross – section of a cylindrical pillar consisting of a solid cylinder A and a hollow cylinder B each of height 2.5 m and made of the above mentioned material is shown in the figure. Radius of A is 10 cm and the external and internal radii of B are 10 cm and 7 cm respectively. (The diagram is not drawn to the scale)

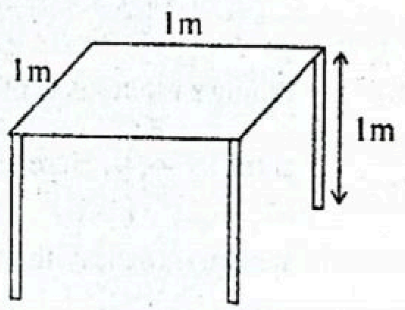


(a) What is the compression of the pillar when a load of 1×10^5 N is supported by it?

(b) What is the maximum load that can be supported by the pillar without breaking it?

2002 A/L

4) One leg of a four- legged square stand used to place a sensitive instrument is found to be longer by 0.1 mm than the other three legs of length 1.0 m each causing the, stand to wobble slightly. Each cylindrical leg has a cross sectional area of 1.0 cm^2, and they are made of a material having Young's modulus 2.0×10^{11} Nm^{-2}. The stand-top consists of a uniform square board of side 1.0 m long. The four legs are fixed at the corners of the board as shown in the figure. Assume that the stand has negligible mass.



(i) Keeping a weight at a suitable location on the stand, the stand-top can be made horizontal by compressing the long leg alone so that the stand no longer wobbles.

(1) Where do you keep the necessary weight on the stand?

(2) Find the weight thus required.

(ii) Instead of the weight used in (i) another weight of 4000 N is placed on the stand to compress all four legs so that the stand-top becomes horizontal and the stand does not wobble.

(1) Find the decrease in length of each leg.

(2) Find the reaction on each leg by the floor.

(3) Where do you keep the weight?

2007 A/L

5) An acrobat A stands on one hand as shown in figure (a). Assume that the bone of the upper arm U of the acrobat is a solid cylinder with an inner cylindrical cavity. When not subjected to stress the length of this cylinder is 0.3 m. Its outer radius and that of the inner cylindrical cavity are 10^{-2} m and 4×10^{-3} m respectively. The weight of the acrobat excluding the arm is 600 N. Young's modulus and the breaking stress of a human bone are 1.4×10^{10} Nm^{-2} and 9.0×10^{7} Nm^{-2} respectively.

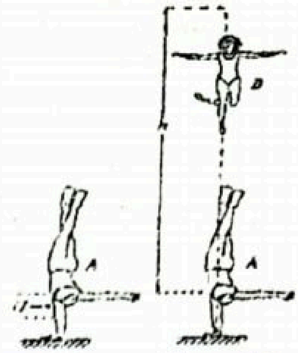


Figure (a) Figure (b)

(i) What is the compressional strain of the upper arm bone when he is standing as shown in figure (a)? By how much is the bone compressed?

(ii) What is the elastic energy stored in a unit volume of the bone?

(iii) Starting from rest, another acrobat B of mass 50 kg now jumps vertically on to A from a height h as shown in figure (b). After landing on the shoulder of A, which is right above his upper arm bone, B takes a time of 0.02 s to come to rest.

(a) Once landed on A and came to rest, what is the change in momentum of B in terms of h?

(b) Find the average value of the force in terms of h exerted on A by B due to the change in momentum.

(c) Calculate the maximum height from which B can jump on to A without breaking the upper arm bone of A. (Assume that Hooke's law is applicable until the breaking stress is applied.)

2009 A/L

6) a) Young's modulus E of a material in the form of a wire is given by $\frac{F/A}{\Delta l/l}$. Here all the symbols have their usual meaning. Identify the terms F/A and $\Delta l/l$ in the expression.

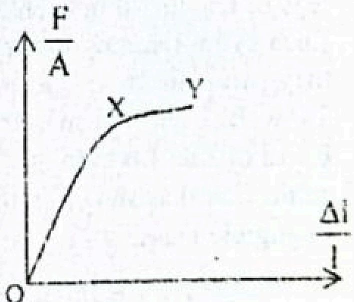


Figure (1)

b) Figure (1) is a characteristic curve showing the elastic behaviour of a material identify the points X and Y marked on the curve.

c) Two uniform nylon strings of length l (=10 cm) $2l$ (=20 cm) of similar area of cross – section A are separately fastened to a rigid oval shaped frame as shown in figure 2(a). Both strings are just stretched with negligible tension. The strings lay perpendicular to each other and run just touching each other.

Now a force is applied to the point of contact of the strings and perpendicular to the plane containing the strings as shown in figure 2(b).

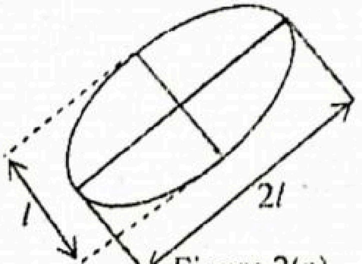


Figure 2(a)

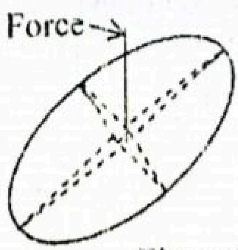
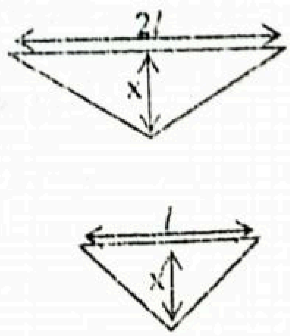


Figure 2(b)



If x is the depression of the point of contact of strings (see figure 2(b)) due to the application of force,

i) Write down, expression for the increase in length of the two strings in terms of x and l.

ii) Derive expressions for the tensions of the two strings in terms of E, A, l and x, where E is the Young's modulus of the material of the nylon strings.

iii) If x = 0.5 cm, substitute the values given for l and x, and hence show that the tension of the shorter string is higher than that of the longer string.

[When x = 0.5 cm and l = 10 cm, take $\sqrt{x^2+l^2} = 10.0125$ cm, and $\sqrt{x^2+\frac{l^2}{4}} = 5.025$ cm]

d) i) Explain qualitatively how the two tensions behave as the applied force to the point of contact of strings is increased.

ii) Draw rough sketches of the tension (T) versus extension Δl curves for both strings on the same graph and label them.

iii) Suggest a method which enables both strings to reach the condition depicted by point X shown in figure (1) simultaneously.

2012 A/L.

7) Figure (1) shows the stress – strain curve for a uniform steel rod. Identify the points A, B and C. An underground storage (S) of length 150 m, and width 6 m is to be constructed at a depth of 20 cm from the ground level. Figure (2) shows the side view and figure (3) shows the front view of the storage. The weight of the soil existing above the roof of the storage is to be supported entirely by 30 cm × 30 cm square steel columns (C). The soil has a uniform density of 3.0×10^3 kgm^{-3}

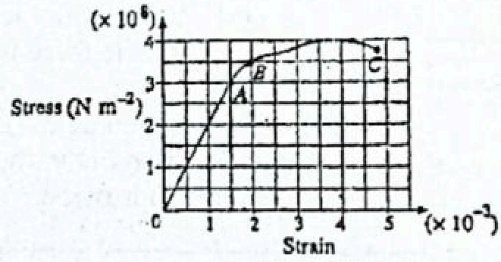


Figure (1)

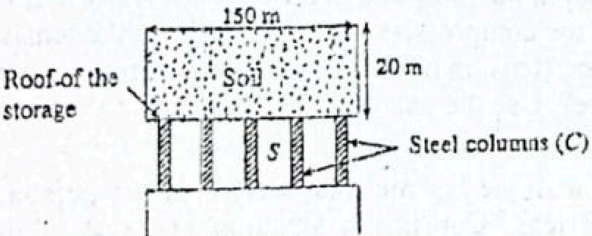


Figure (2)

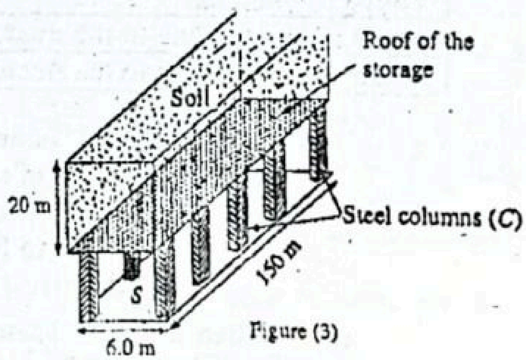


Figure (3)

a) i) Calculate the total weights of the soil that the column must support.

ii) What is the number of columns needed to keep the compressive stress on each column at 2×10^8 Nm^{-2} ? Assume that the weight of the soil is equally distributed among the columns. Neglect the mass of the roofing material.

b) i) Determine the Young's modulus of steel from the curve given in figure (1) above.

ii) If the height of a steel column is 4.995 m what was its original uncompressed heights ?

(iv) Consider a person having all body dimensions doubled including all bones compared to an ordinary person. Let the mass of such a person 600 kg. If the scaled-up person now stands on one leg, does he feel uncomfortable? Justify your answer. Assume that the elastic characteristics given in the table above remain unchanged for the situation.

2018 A/L

9) (a) Concrete is a hardened mixture of cement, sand, gravel and water. Reinforced concrete structures are structures composed of concrete and steel bars. All rigid bodies, such as steel and concrete are elastic to some extent. Concrete is strong under compression but weak under extension while steel is strong under both situations. As a combination, concrete mainly resists compression, and steel bars mainly sustain the tension.

Consider a plain concrete beam having rectangular cross-section, and **without** steel bars, kept on two supports, and subjected to a load W as shown in figure 1(a). Under this situation the bottom part of the beam will experience an extension while the top part will experience a compression as shown with dotted lines.

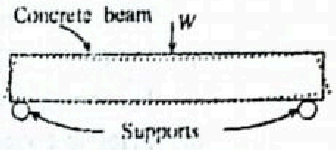


Figure 1(a)

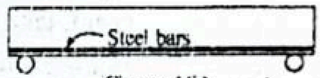


Figure 1(b)

(i) Which side of the plain concrete beam (top or bottom) is most vulnerable to crack under the load, W

(ii) To improve the situation shown in figure 1(a), steel bars are inserted closer to the bottom of the concrete beam at the production stage as shown in figure 1(b). Based on the information given at the beginning of the question, explain, how this improves the load bearing capacity and prevents cracking of the concrete beam.

(b) The tensile stress $\left(\frac{F}{A}\right)_S$ - strain $\left(\frac{\Delta l}{l}\right)_S$ relationship for mild steel (S) can be modelled, as shown in figure 2(a). Even though concrete is a brittle material, the tensile stress $\left(\frac{F}{A}\right)_C$ - strain $\left(\frac{\Delta l}{l}\right)_C$ relationship of the concrete (C) under tensile force can also be modelled as shown in figure 2(b). In reinforced concrete, steel bars are well bonded to concrete, thus they can jointly resist external loads together until concrete cracks.

When the curve reaches the point P shown in figure 2(b), the concrete will crack.

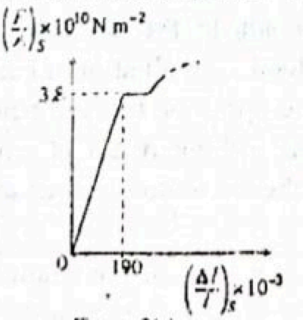


Figure 2(a)

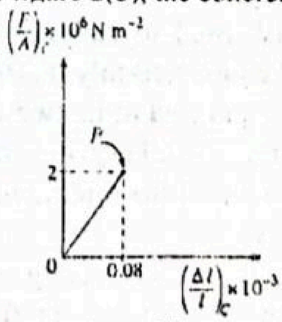


Figure 2(b)

Using the figures 2(a) and 2(b),

(i) calculate Young's modulus of mild steel E_S.

(ii) calculate Young's modulus of concrete E_C.

(c) Figure (3) shows a reinforced uniform concrete beam of length l kept on a rigid horizontal surface. The beam is reinforced with concrete and identical four uniform cylindrical mild steel bars each of length l. The tensile stress-strain relationships corresponding to the concrete and the steel used are given in figures 2(a) and 2(b) respectively. Assume that the beam is subjected to total tensile force of F_t applied uniformly throughout the area of cross-section of the beam, and mild steel bars and concrete produce same extension Δl under the tensile force.

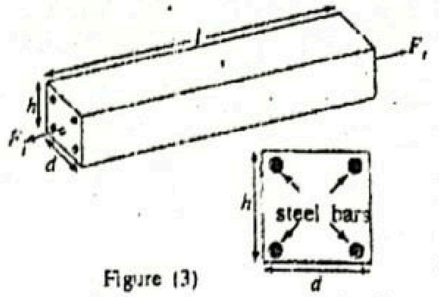


Figure (3)

i) Write down an expression for the tensile force (F_C) on concrete in terms of E_C, area of cross-section of the concrete A_C, l and Δl.

ii) Write down an expression for the tensile force (F_S) on the four mild steel bars in terms of E_S, total area of cross-section of the four mild steel bars A_S, l and Δl.

iii) Prior to concrete cracking, if the total tensile force (F_t) is carried by both concrete and the steel, obtain an expression for the total tensile force F_t on the reinforced concrete beam.

iv) The area of cross-section A of the reinforced concrete beam is dh. See figure (3). For the beam, take $l = 2000$ mm, radius of a cylindrical mild steel bar $r = 6$ mm, $\Delta l = 0.1$ mm, $d = 150$ mm and $h = 250$ mm.

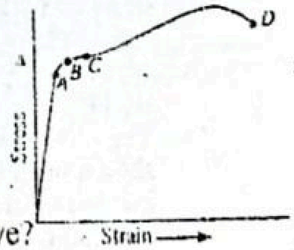
(1) Physically under what condition the expression obtained in (c)(iii) above is valid? Use the data provided above for the reinforced concrete beam and show that the expression obtained in c(iii) is physically valid for the beam.

(2) Calculate the value of F_t. (For your calculation, if $\frac{A_S}{A} \leq 3\%$ then take $A_C = dh$, otherwise take $A_C = dh - A_S$. Take π as 3.)

v) Calculate the minimum tensile force which cracks the reinforced concrete beam.

2020 A/L-7

10) a) i) Stress – strain curve for a metal wire is shown in figure (1). Identify the characteristic points A, B, C and D.



ii) If the wire is stretched up to the value depicted by point C and released, what will happen to the wire?

iii) What is represented by the area under the stress – strain curve?

b) Iron beams are used to support heavy loads in the construction of structures and buildings. When a uniformly distributed load is applied on a beam with a rectangular cross section supported at its two ends, the upper part of the beam is compressed and becomes shorter. Similarly the lower part of the beam is elongated and becomes longer. The length of the middle layer of the beam does not change and it is known as the neutral axis.

The distribution of forces acting on the upper part of the beam of thickness d is illustrated in figure (2). The figure is now drawn to scale. Copy this diagram on to your answer sheet and draw the distribution of forces acting on the lower part of the beam.

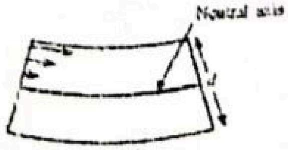


Figure (2)

c) The lower section of the beam in figure (2) is shown in figure (3). The radius of curvature of the neutral axis is r which subtends an angle α (in radians) at the center O. The length of the neutral axis of the beam is l.

i) Write down an expression for l in terms of r and α.

ii) Write down an expression for l' in terms of r, d and α. Here l' is the length of the bottom layer (B) of the lower section of the beam.

iii) Show that the average value of the strain existing on the lower section of the beam is given by $\frac{d}{4r}$.

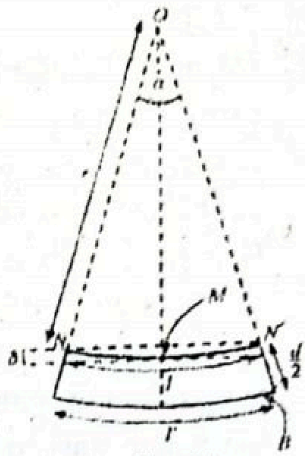


Figure (3)

d) i) What is the force acting along the neutral axis (NN')?

ii) If the average value of the tensile force acting on the lower section of the beam is F, what will be the force acting along the bottom layer (B) of the lower section of the beam?

iii) If the width of the beam is w and the Young's modulus of iron is Y, show that force F is given by $F = \frac{wd^2Y}{8r}$.

iv) When the lower section of the beam is subjected to a average tensile stress of 1.0×10^8 Nm^{-2}, determine the value of radius r. Young's modulus of iron, Y $= 2.0 \times 10^{11}$ Nm^{-2}; d = 20 cm.

v) If $l = 5.0$ m, determine α in radians.

vi) Using $\cos(\frac{\alpha}{2}) = 0.9997$, calculate the depression δ of the midpoint (M) of the neutral axis of the beam.

e) Figure (4) shows a rectangular beam and an I (or H) – shaped beam made out of iron. In the construction field, I-shaped beams are generally used instead of rectangular beams. Giving reasons state the advantage of this.

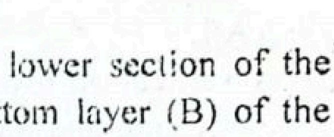


Figure (4)