Advanced Level

PHYSICS

Essay

Questions

Classified Essay - Book 1

* Mechanism
* Oscillation and waves – Sound
* Oscillation and waves – Optics

1990–2020

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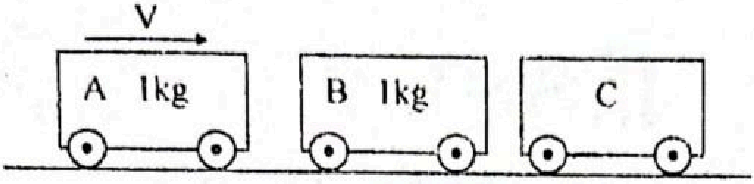
1993 A/L

1) (a) Distinguish between elastic and inelastic collisions between two bodies. Give an example for a completely inelastic collision.



Three trolleys A, B and C of mass 1 kg, 1 kg and M respectively are kept at rest on frictionless horizontal rails, As shown in the figure the trolley A is projected with a velocity V towards the trolley B. Assuming all collision taking place to be elastic.

(i) Show that when the trolley A collides with B, A becomes stationary and B takes off with velocity V.

(ii) State how many subsequent collisions will occur and find the final velocities of all trolleys in terms of V if M = ½ kg.

(iii) State what happens if M = 2 kg and find the final velocities of all trolleys in terms of V.

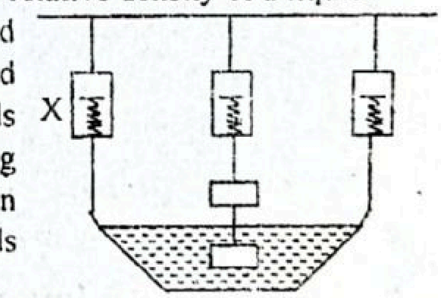
(iv) If instead the rails have friction are the conservation laws that you have used still valid? Explain your answer.

2) State Archimedes principle

Explain why it is easier to float a uniform cylindrical body horizontally in a liquid than vertically. How can this cylinder be made to float vertically?

Describe how such a cylinder can be used to determine the relative density of a liquid.

A pan of water is suspend from two spring balances. X and Y as shown in the figure. A block of brass is suspended from a third balance Z. X and Y read 1 kg each and Z reads 1.2 kg. When the block is gradually lowered by increasing the length of the string so that it is completely immersed in water as shown by the dashed lines, the balance Z reads 0.80 kg. Find the new readings of X and Y.



If the brass is made of copper and zinc of densities 9×10^3 kg m^{-3} and 7×10^3 kg m^{-3} respectively, find the mass of zinc in the block. The density of water is 10^3 kgm^{-3}

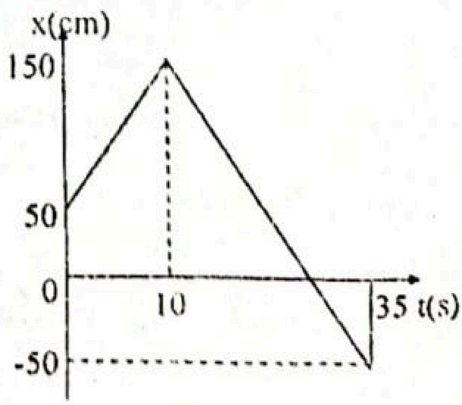
1994 A/L

3) (a) Figure shows a displacement (x) -- time (t) curve of an object which moves in a straight line on a horizontal table. Mass of the object is 0.5 kg.

(i) Find the initial and the final velocities of the object.

(ii) (a) Draw corresponding velocity-time curve for the whole journey of the object.

(b) Determine the total distance traveled by the object.



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(c) Explain what happens to the motion of the object at t = 10 s. Give a practical example where similar changes that occur at t = 10 s can be observed.

(iii) Suppose the object experiences a constant frictional force exerted by the table after 35 s, and it comes to rest in further 2s.

(a) What is the magnitude of the frictional force acting on the object?

(b) Calculate the coefficient of kinetic friction between the object and the table.

(b) The present electrical energy consumption in Sri Lanka is 3.0×10^9 kWh per year.

(i) Calculate the above energy consumption per year in Joules

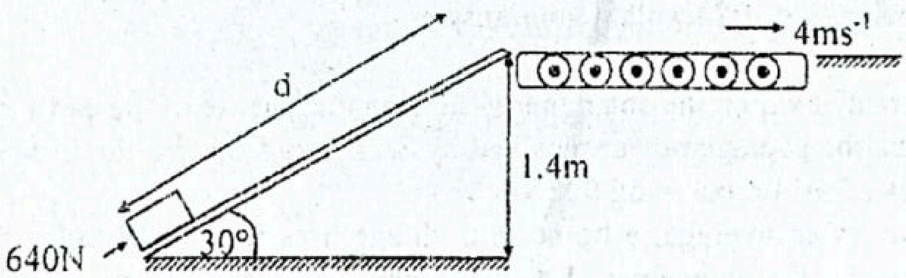
(ii) Calculate the minimum mass of water needed per year to generate the above amount of electricity in a hydro-power plant, if the water falls from a vertical height of 200 m. State clearly the assumption that you have made to arrive at the answer.

(iii) Taking the rate of flow of water to be constant throughout the year, determine the force exerted by falling water on a turbine blade in the generator. Assume that the water strikes the blade perpendicular to its surface and then flows along the surface without any recoil.

(iv) The Ceylon Electricity Board estimates that the energy demand for electrical power will increase to 7.5×10^9 kWh per year in the year 2000. The Board plans to meet this increase in energy demand by operating coal powered thermal power stations. Calculate the mass of coal needed per year to generate the extra amount of electrical energy. Assume that a coal power station operates with an overall efficiency of 40%.
(1 kg of coal on burning gives 4.5×10^5 kJ of energy.)

1995 A/L

4) a)



A box of mass 100 kg is to be raised to a vertical height of 1.4 m by pushing it up an inclined plane and then to be transferred over to a horizontal moving conveyor belt, as shown in the figure, it is found that a minimum force of 640 N is necessary to move the box along the inclined plane which makes an angle 30° with the horizontal.

(i) What is the total work done by the above applied force in pushing the box up the inclined plane?

(ii) What is the corresponding increase in the potential energy of the box?

(iii) If the value obtained in (i) is different from (ii), explain the reason for it,

(iv) Calculate the coefficient of friction between the box and the inclined plane

(v) At the top of the inclined plane the box is transferred instantaneously with a negligibly small speed on to the belt which is moving horizontally at a constant speed 4 ms^{-1}. The box acquires the speed of the belt 2 s at after it touches the belt.

(a) What is the change in momentum of the box along the horizontal direction?

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(b) Calculate the magnitude of the force acting on the box during the 2 s, in order to acquire the above momentum. Explain how dose this force originate.

(c) What is the magnitude of the external force which should act on the belt in order to keep it moving at constant speed during the above mentioned 2s. Where does this force come from?

1996 A/L

5) (a) An ice skater A of total mass 65 kg (including his helmet) slides freely on a frictionless frozen lake in a straight line with a velocity of 2 ms^{-1}. While moving, A throws his helmet weighing 5 kg horizontally with a velocity of 4 ms^{-1} in a direction perpendicular to his direction on motion.

(i) Find the magnitude of the resultant velocity of A after throwing the helmet.

(ii) Another skater B of mass 45 kg sliding freely in the opposite direction on a nearby parallel course with a velocity 1 ms^{-1} catches the helmet thrown by A. After catching the helmet, find the new velocity of B in.

(a) the original direction of motion of B.

(b) the direction perpendicular to the original direction of motion of B.

(iii) Calculate the total kinetic energy of the helmet and the skater B just before B catches the helmet.

(iv) Calculate also the total kinetic energy of the helmet and the skater B after B catches the helmet.

(v) Explain briefly why the two values calculated in (iii) and (iv) above are different.

(vi) After some time the helmet falls off freely from the skater B. What will happen to the velocity of B? Explain your answer.

(b) The average rate at which the solar energy falls on the surface of the earth is 1 kWm^{-2}

(i) Calculate the average power received by Sri Lanka from the sun in MW.

Area of Sri Lanka = 65 000 km^2

(ii) Suppose, in an average, a home in a village uses five 40 W bulbs for 3 hours daily, and also consumes 1.4 kW-hours daily to operate other electrical appliances. Calculate the daily energy requirement for 100 such homes in a village.

(iii) A project is planned to use solar panels to generate the energy requirement calculated in (ii). If solar panels convert sunlight to electricity with 10% efficiency, and the average power generation period of solar panels is 5 hours per day calculate the total area of the solar panels needed to fulfill the energy requirement of the village. Assume that the solar panels are placed parallel to the earth surface and capable of delivering electrical energy to bulbs and other appliances with 80% efficiency.

(iv) At present the total electrical power generation capability in Sri Lanka stands at 1400 MW. If this value is to be increased to 2 000 MW using solar panel based power generators, calculate the total area of the solar panels needed for this purpose.

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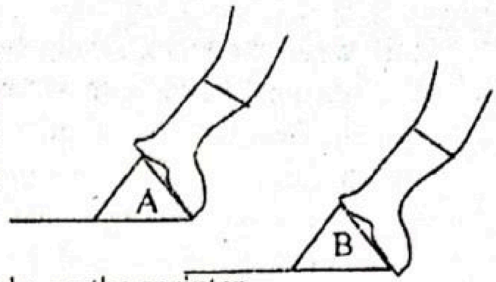
1997 A/L

6) A 70 kg sprinter running a race of 100 m pushes on the starting block (B) for 0.2 s and leaves them with a speed of $5ms^{-1}$. He then accelerates for a further 5 s so that his speed becomes $12\ ms^{-1}$ and continues with that speed until the finish line.



(i) Find the reaction force exerted by the starting blocks on the sprinter.

(ii) Calculate the distance covered by the sprinter by the time he attains the maximum velocity of $12\ ms^{-1}$.

(iii) What is the mechanical work done by the sprinter during the accelerating period of 5 s?

(iv) Find the time taken by the sprinter to complete the race.

(v) In this race another sprinter who took a better start by achieving an initial velocity of $5.4\ ms^{-1}$ in the same period of 0.2 s as above, however, spent 5.4 s to reach his maximum speed of $12\ ms^{-1}$. Calculate the time at which the former sprinter overtakes the latter.

[Hint: overtaking takes place during the accelerating period of the former sprinter]

7) An empty boat floats in water with 10% of its volume submerged, and when it is loaded with 1200 kg, the volume submerged will increase to 70% of the total volume.

(i) Calculate the mass of the empty boat.

(ii) If a leak develops and water starts to enter the boat which is loaded with 1200 kg at a constant (average) rate of 100 kg per minute, for how long the boat can float before sinking?

(iii) What is the minimum force required to lift the sunk boat (without the load) up to the surface of water. Average density of the material of the boat is $2500\ kg\ m^{-3}$ and the density of water is $1000\ kg\ m^{-3}$

(iv) The mended boat with another load of 1200 kg on board, while sailing, suddenly enters a region where water is uniformly mixed with tiny air bubbles. If the average volume of an air bubble is $1\ mm^3$ and the air bubble concentration is $3.5 \times 10^8\ m^{-3}$, find the effective density of water. Neglect the mass of air. Hence show that the boat will sink.

(ii) Use the effect described in (iv) to explain the danger in the following act. 'There is a deep pool at the foot of a tall water fall. A person starts to swim closer to the foot of the fall'!

1998 A/L

8) A block of mass 1.4 kg is hung by a light inextensible string. A bullet of mass 0.1 kg moving horizontally with a velocity of $60\ ms^{-1}$ collides with the block and gets embedded to the block.

(i) What is the kinetic energy of the bullet before the collision?

(ii) Calculate the percentage loss of the kinetic energy of the system due to the collision. Does this loss imply that the law of conservation of energy is violated here? Explain your answer.

(iii) Calculate the maximum height to which the block is raised after the collision.

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(iv) When the block swings back to its original position for the first time, a second identical bullet with same velocity hits the block and gets embedded. What is the final velocity of the block just after the collision?

(v) If the above string is replaced by a light elastic string repeat the calculation in (iii) above, for the collision of the first bullet. Extension of the string before the collision is 0.2 m, and the extension when it is at the maximum height is 0.1 m.

9) Write down the Bernoulli equation for a fluid flow, clearly identifying the symbols used. What quantity does each term in this equation represent.
State the conditions under which the Bernoulli equation is valid
During heavy wind, sometimes roofs of closed buildings are blown off. Use the Bernoulli equation to explain this phenomenon.

(i) A narrow stream of gas blows out of a gas jet in the horizontal direction. In order to measure the speed of gas at the outlet of the jet, a student uses a U tube which is open at both ends and containing an oil. When the U tube is held vertically and close to the outlet so that only one of the ends is in the stream of gas, he notices a difference of 2.4 cm between the oil levels of the U tube. Find the speed of gas at the outlet of the jet.

(ii) If the area of cross – section of the gas stream at the outlet is 10^{-4} m^2, find the rate of mass flow of the gas in the stream

(iii) Calculate the power of the gas jet.

Density of gas	=	1.2 kg m^{-3}
Density of oil	=	800 kgm^{-3}

1999 A/L

10) Read the following passage carefully and answer the questions give below .

Divers, acrobats, and ballet dancers perform many graceful rotational movements. All these movements can be explained in terms of the physical concepts related to rotational motion.

The rotation of a human body can be related to three mutually perpendicular axes which pass through the centre of gravity (G) as shown if fig (1). Rotation about the y-axis is called a somersault, about the z-axis it is a twist, and about the x-axis it is a pinwheel motion. While performing the twist the body rotates in the plane xy.

The moments of inertia (I) about these axes depend on the positions of the arms and legs. In general I_z is small than I_x or I_y . For an average person standing as depicted in figure (1), the values are I_z = 3.4 kg m^2, I_x = 19.2 kg m^2 and I_y = 16.0 kg m^2. In the "truck" position shown in figure (2), the values are I_z = 2.0 kgm^2 and I_x = I_y = 4.0 kg m^2.

The most obvious way a diver can acquire a somersault motion on take off is by using a diving board. Fig (3) shows how the person can acquire angular momentum about the axis y. He merely leans forward as he jumps. The normal reaction N due to the board produces a torque about his centre of gravity.

Now consider how a person acquires somersault motion while he is in free fall. The body is held rigid and the raised arms are brought rapidly forward in a 'arm swing' motion as in fig (4). As the arms are brought down, the body will rotate in the opposite sense. The axis of rotation is at the shoulders. The somersault will continue as long as the arms

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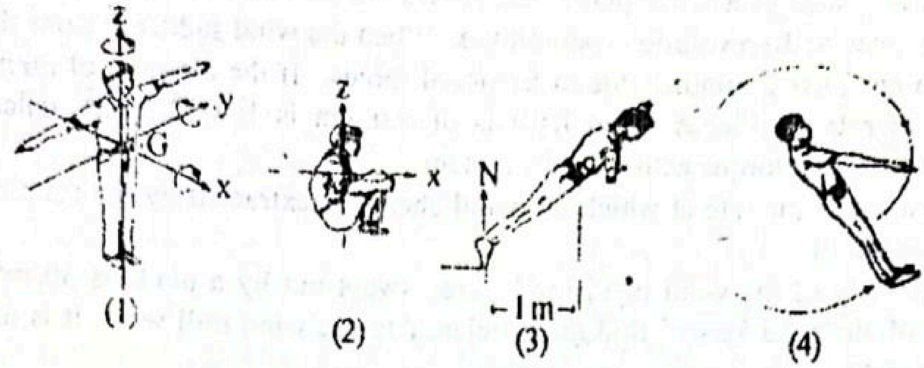
perform this 'arm swing' motion. However, the rotation of the body is slow compared to the rotation of arms.



(i) Name the plane of rotation of the person in figure (1) when he performs a somersault.

(ii) Mass of an object measures the inertia for linear motion. What does the moment of inertia of an object about a given axis measure?

(ii) How can a person change his moment of inertia about a given axis on his own?

(iii) For the person shown in Fig (1) I_z is smaller than I_x or I_y. What is the reason for this?

(iv) The person shown in Fig(1), makes a somersault with angular velocity 2.0 rad s^{-1}. While in rotation he changes to the position shown if fig (2)

(a) calculate the new angular velocity of the person

(b) calculate the change in rotational kinetic energy of the person. How do you account for this change?

(v) If the mass of the person is 60 kg determine his initial angular acceleration about his centre of gravity when he takes off the board as in Fig (3)

(vi) When the arms are swinging rapidly as in Fig (4), what is the reason for the slow rotation of the body?

(vii) Is the angular momentum of the person in Fig (4) conserved about an axis going through his shoulders? Give reason for your answer.

(viii) We use this 'arm swing' technique instinctively when we are about to slip on a wet floor. If our feet start to slip forward, a rapid arm swing motion opposite to that in fig (4) is executed. Briefly explain the reason for this

2000 A/L

11) (i) Wind blows along a horizontal direction in an open space at a constant velocity v. If the density of air is ρ write down an expression for the kinetic energy (E) per unit volume of a moving column of air.

(ii) The kinetic energy carried by the wind can be extracted through the rotating blades of a wind mill and subsequently converted to useful energy. Consider a situation where the wind is blowing normal to the plane of rotation of the blades of a wind mill. The area swept out by a rotating blade is A. Assuming that all the kinetic energy of the wind blowing through a cross-sectional area A could be extracted by the blades, show that the rate at which the wind energy is transferred to the wind mill is, $\frac{1}{2}\rho A v^3$

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(iii) When a certain wind mill is in the free running state (i.e. when it is not coupled to any other device such as a water pump) the blades are found to be rotating at a constant angular speed of 30 revolutions per minute. When the wind suddenly stops the blades come to rest after 2 minutes due to frictional forces. If the moment of inertia of the system of rotating blades about its axis of rotation is 10,000 kgm^2 calculate the average frictional torque acting on the system.

(iv) Hence calculate the rate at which the wind energy is extracted by the rotating blades of the wind mill

(v) If the velocity of the wind is 10 ms^{-1}, area swept out by a blade is 30 m^2 and the density of air is 1.3 kg m^{-3} find the efficiency to the wind mill when it is in the free running state.

2001 A/L

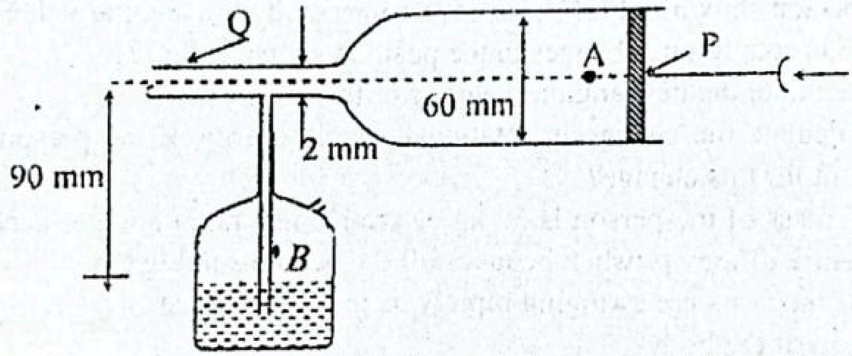
12) Bernoulli's equation for a fluid flow can be written as, $p + \frac{1}{2}\rho v^2 + h\rho g$ = constant.

Where all the symbols have their usual meaning

(a) (i) State the conditions under which the Bernoulli's equation is valid

(ii) Show that the above equation is dimensionally correct.

(b)



The insecticide sprayer shown in the figure has a pump with a diameter of 60 mm. The diameter of the outlet tube Q is 2 mm and the insecticide level is 90 mm below the tube. Assume that the pressure at A is same as that at B and, air behaves according to all the conditions that you have mentioned in (a) (i)

(i) Calculate the minimum speed with which the piston P of the pump should be pushed if the air jet in the tube Q is to contain insecticide.
(Take the densities of insecticide and air to be 10^3 kg m^{-3} and 2 kg m^{-3} respectively)

(ii) If the net resistive force acting on the piston of the pump is 20 N, determine the force that has to be applied on the piston, in order to maintain the sped calculated above.

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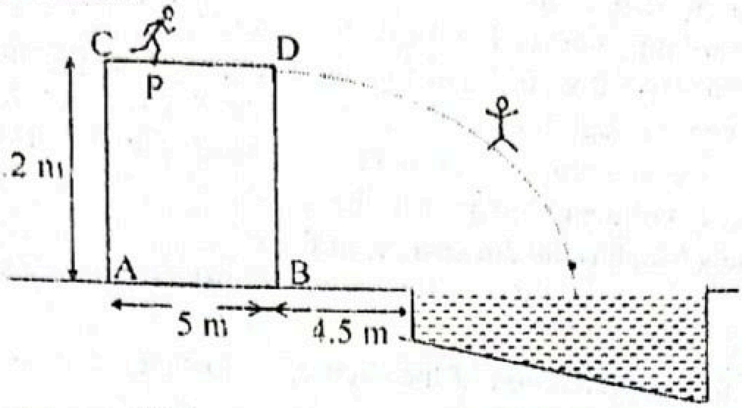
13) As shown in the figure a fun game involves running along the top of a platform P and falling into a pool below



A student of mass 50 kg accelerates uniformly from rest at one end (C) of the platform to the other end (D) and leaves the platform in the horizontal direction at a speed of 5 ms^{-1} without any rotational motion.

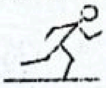
The length of the platform is 5m. (Neglect air resistance)

i) (a) Calculate the acceleration of the student while he is running on the platform

(b) How long does he take to reach the other end (D) of the platform?

(c) State clearly how does the student obtain the required external force in order to achieve his acceleration.

(d) Mark clearly the forces acting on the student while he is running on the platform

(copy the diagram given here  on your answer sheet for this purpose)

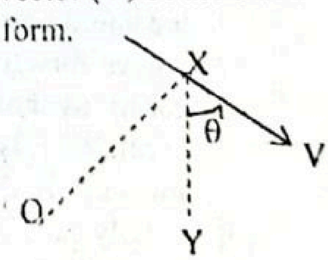
ii) (a) How long does he take to touch the water after leaving the platform?

(b) Determine the horizontal distance form the point B to the point at which he lands on water.

(c) Mark clearly the force / forces acting on the student while he is falling in air (copy the diagram given here  on your answer sheet for this purpose)

iii) Sketch the velocity (V) – time (t) curve for the horizontal component of the velocity of the student from the start (C) until he touches water.

iv) The figure shown the direction of the instantaneous velocity vector (V) of the student when he has fallen a vertical distance of 1.25 m from the platform.



(a) Calculate the magnitude, and direction (i.e angle θ between V and vertical line XY) of velocity V.

(b) At this instant the motion of the student can be treated as a part of a circular motion around a point O. Determine the centripetal acceleration of the student at this moment.

(c) Hence calculate the radius of the corresponding circle.

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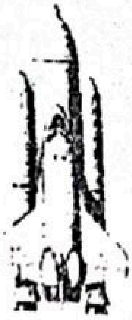

14) A space shuttle, when it stands on the launching pad, has a mass of 2.0×10^6 kg. The upward thrust needed to move the shuttle is 3.0×10^7 N, which is achieved by burning 3.0×10^3 kg of fuel per second and expelling the hot gas thus produced, through the nozzle at the bottom. The force of upward thrust is given by the product of the rate (M) at which fuel is burnt and the exhaust velocity (u) of the gas relative to the shuttle.

i) Show that the product Mu, has the dimensions of force.

ii) (a) What is the initial acceleration of the shuttle just as it begins leaving the launching pad?

(b) Assuming that the acceleration of the shuttle is constant, determine the velocity of the shuttle 30 s after take off?

iii) (a) Calculate the exhaust velocity (u) of the gas relative to the shuttle

(b) What is the exhaust velocity of the gas relative to the earth 30 s after take off?

iv) A student states that the shuttle cannot accelerate if there is no atmosphere outside. Is this statement correct? Explain your answer?

v) (a) in reality the acceleration of the shuttle increases as it burns fuel, even though the upward thrust on the shuttle is constant. Explain this statement.

(b) Sketch the velocity(v) time (t) curve for the shuttle corresponding to the situation in (v) (a) above

vi)

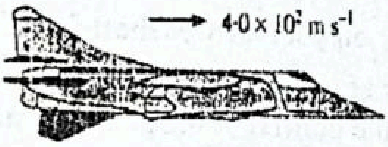

Figure (A)

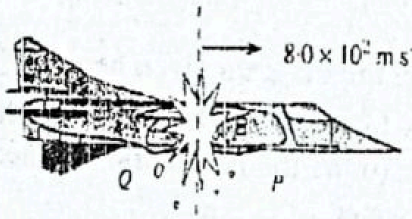

Figure (B)

(a) Consider a situation where the shuttle is moving horizontally close to the earth surface as shown in figure (A) with a velocity of 4.0×10^2 ms^{-1}. The mass of the shuttle at this instant is 1.0×10^5 kg. Unfortunately, due to an internal explosion the shuttle breaks into two pieces (P and Q) with equal masses. If the piece P moves forward horizontally with a velocity of 8.0×10^2 ms^{-1} (relative to the earth) as shown in figure (B), determine the velocity of piece Q relative to the earth. What is the velocity of Q relative to P? Assume that there is no loss in the mass of the shuttle due to explosion.

(b) Briefly state the subsequent motion of pieces P and Q after the explosion as seen by an observer on the earth.

(c) If the explosion lasts for 0.2 s what is the average value of the force exerted on each piece due to the explosion?

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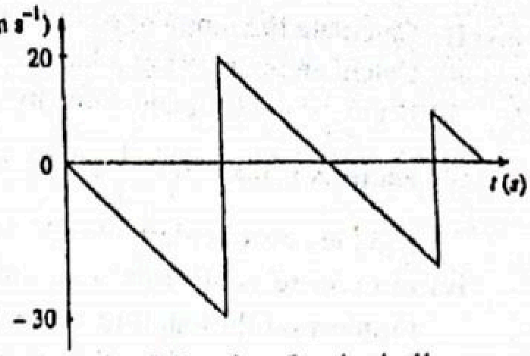
2004 A/L

15) A small ball of mass 0.1 kg is dropped from rest at t = 0 onto a horizontal floor. The ball was initially at a height H from the floor, and it bounces back vertically after each and every collision. Figure shows a portion of the velocity (v) – time (t) graph of the ball.



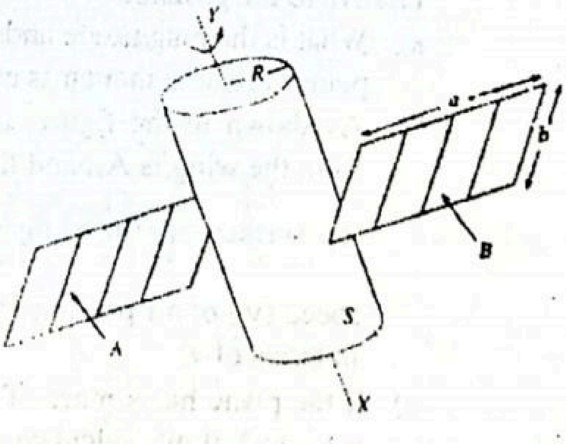
i) Neglecting air resistance and the up thrust, calculate the following for the ball
   (a) The initial height H.
   (b) The change in momentum of the ball, and the momentum transferred to the floor at the first collision.
   (c) The value of t at which the second collision occurs.

ii) If the collision between the ball and the floor is perfectly elastic, draw the v-t graph for this motion.

iii) A particle of mass 6×10^{-26} kg in an empty cubical box of side length 1 m is made to move back and forth, while making collisions with two opposite walls of the box normally. The collisions between the particle and the walls are perfectly elastic, and the speed of the particle is 2×10^{3} ms⁻ (Assume that the gravitational force on the particle is negligible.)
   (a) Calculate the rate at which the particle collides with one of the two walls.
   (b) What is the rate at which the particle transfers momentum to that wall?
   (c) Suppose that the box contains 2×10^{23} such particles performing the same motion mentioned above. Assume also that these particles do not make collisions among themselves, and that the collisions are uniformly distributed over the entire area of the wall. Calculate the pressure exerted by the particles on one of the two walls.

2005 A/L

16) Figure shows a satellite having a cylindrical body S and two identical solar panels A and B. This satellite is moving in the space where gravity is negligible and rotates about the axis of the cylinder XY with an angular velocity of 6 revolutions per minute. The plane of the solar panels is perpendicular to the XY axis of the cylinder. Radius of the cylinder $R = 0.4$ m and moment of inertia of the cylinder about XY axis $I = 6 \text{kg m}^2$ For each solar panel, mass $m = 2$kg, length $a = 1.2$m and width $b = 0.6$ m . The moment of inertia of each solar panel about XY is given by



$$m\frac{(a^2+b^2)}{12}+m\left(R+\frac{a}{2}\right)^2$$

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i) Calculate the moment of inertia of the satellite about XY.

ii) Calculate the rotational kinetic energy of the satellite.

iii) If the two solar panels are folded so that the new moment of inertia of each panel about XY becomes $\frac{1}{4}$ of the previous value, calculate the new moment of inertia and the new angular velocity of the satellite about XY.

iv) In order to control the rotation of the satellite a mechanism is available to apply a torque τ on the satellite along XY. This mechanism does not change the moment of inertia of the satellite.

a) If the angular velocity of the satellite has to be brought back to its original value, from the value calculated in (iii) above, by maintaining a uniform angular deceleration for a period of 5 minutes, calculate the magnitude of the angular deceleration and the torque τ required.

b) Determine the energy required to bring the angular velocity of the satellite back to the original value.

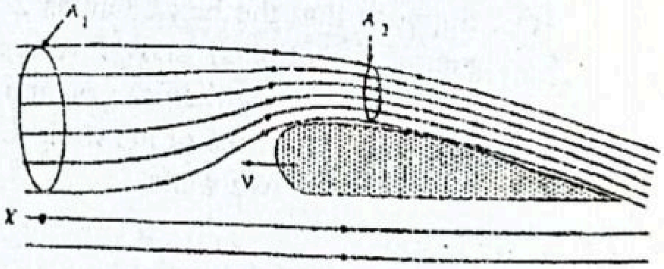
2006 A/L

17) i) Bernoulli's equation for a fluid flow can be written as

$$P + \frac{1}{2}\rho v^2 + h\rho g = \text{constant, where all symbols have their usual meaning.}$$

Applying dimensional analysis only to the term $\frac{1}{2}\rho v^2$ show that it has the dimensions of pressure.

ii) Figure shows a cross section of a wing of an aeroplane which is moving through air horizontally to the left at a constant velocity v relative to the ground.



a) What is the magnitude and direction of the velocity of air at point X relative to the plane? Assume that air is at rest relative to the ground.

b) As shown in the figure, the cross-sectional area of a tube of flow exists away from the wing is A_1, and the corresponding area of the same tube of flow over the top surface of the wing is A_2. If $\frac{A_1}{A_2} = 1.2$, write down an expression for the speed (v') of air passing over the top surface of the wing relative to the areoplane in terms of v.

c) If the plane has a mass of 2.64×10^5 kg and total effective surface area of both wing is 250 m^2, calculate the minimum value of v necessary for the areoplane to just lift-off the ground. (The density of air is 1.20 kg m^{-3})

d) The areoplane starts at rest on the runway and applies a constant horizontal driving force of 6.00×10^6 N from its engines. If the average drag force due to air is 7.20×10^5 N, how far does the aeroplane have to travel along the runway in order to achieve the speed v, calculated in (ii) (c) above?

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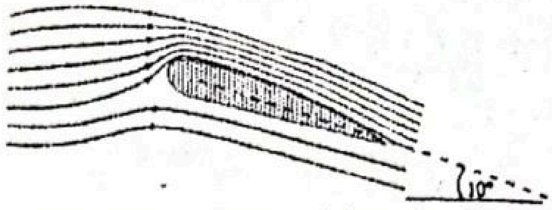
iii) The cross section of a wing of the aeroplane which is moving 10^0 to the horizontal just after take-off, is shown in the figure.



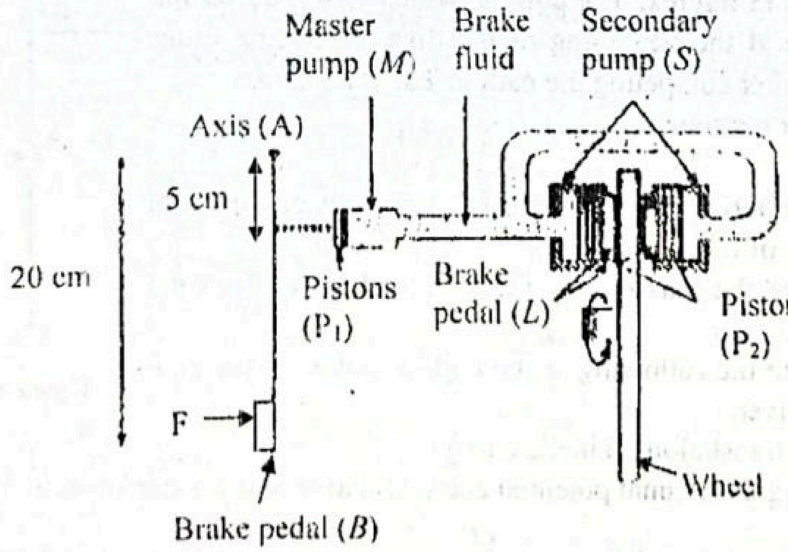
(a) Copy the cross section of the wing on to your answer paper and draw the direction of the net force acting on the wing due to the pressure difference between the bottom and top of the wing.

(b) Now the speed of air over the top surface of wings relative to the aeroplane has increased to 250 ms^{-1}. Assuming the speed of air below the bottom surface of the wings relative to the aeroplane remains the same as in (ii) (*a*), calculate the net vertical lifting force acting on the wings now.

iv) Consider a situation where the plane is moving horizontally at an altitude of 10 km with a speed v_1. If the air is at rest relative to the ground at this altitude also, then the value v_1 should be greater than the value v_1 calculated in (ii) (c) above. Give a reason why this is so. Assume that the mass of the aeroplane has the same value given in (ii) (c) above.

2007 A/L

18) The figure shows a hydraulic braking system which could be used to stop a rotating wheel. A force *F* is applied perpendicular to brake pedal (*B*). The pedal rotates freely about a fixed axis through (*A*) and perpendicular to the plane of the paper as shown in the figure, and causes a force to be applied perpendicularly to the piston (P_1) of the master pump (*M*). The resulting pressure is transmitted by the brake fluid to the two identical pistons (P_2) of the secondary pumps (*S*). Then the brake pads (*L*) attached to the pistons move a little distance and press against both sides of the rotating wheel. Assume that the brake fluid is incompressible. Cross-sectional area of the master piston (P_1) is 1 cm^2, and the cross-sectional area of the secondary piston (P_2) is 3 cm^2.



(i) When a certain force is applied to the master piston it moves a distance of 0.6 cm to the right in this process. How far does a single brake pad (*L*) move?

(ii) If $F = 10$ N

(a) What is the force applied on the piston (P_1) of the master pump? The required distance are marked in the figure.

(b) Calculate the pressure exerted by master piston (P_1) on the brake fluid in Pascal.

(c) Calculate the force exerted on the brake pads due to the pressure created on the secondary piston (P_2).

(d) If the coefficient of dynamic friction between the brake pads and the wheel is 0.5, calculate the frictional force acting on the wheel due to each pad when they are pressed against the wheel.

(iii). Before applying the brakes, the wheel was rotating freely at 600 revolutions per minute. If the distance from the rotating axis of the wheel to the line of action of the frictional force is 5 cm, how long does it take to stop the wheel when brakes are applied with F = 10 N as above? The moment of inertia of the wheel about its axis of rotation is 0.1 kg m^2. Assume that the frictional force remains constant throughout the motion.

How many revolutions does the wheel make before coming to rest? (Take $\pi = 3$)

2008 A/L

19) (a) A diver of mass 50 kg is standing at the end (C) of a 4 m long horizontal diving board (AC) of negligible mass, mounted on two vertical springs, 1 m apart, at A and **B** as shown in figure 1. Find the magnitude and direction of the forces acting on the board at A and B by springs.

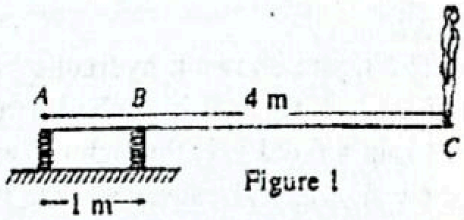


Figure 1

(b) The diver performs a dive. Consider the motion of the centre of gravity (G) of the diver. Its path is indicated by a dotted line as shown in figure2. The point G which is 4 m above the water surface at the beginning of the dive, enters the water surface at Y after competing the path in 2 s. $XY = 2$ m.

(Neglect air resistance.)

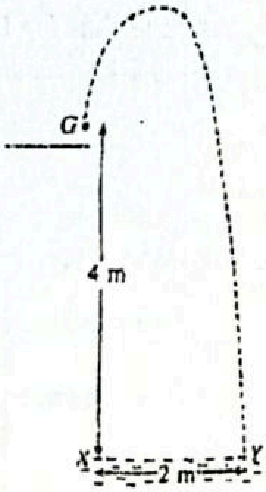


Figure 2

(i) Find the horizontal and vertical components of the initial velocity of G.

(ii) Calculate the maximum height reached by G from the water surface.

(iii) Calculate the following at the highest point of the path of the diver.

(1) The translational kinetic energy.

(2) The gravitational potential energy relative to the water surface.

(c) Diver also performs rotational motion about an axis (take as OP into the paper) passing through G. He controls his rotational motion by bending/extending his body to change the moment of inertia of the body. During the first 0.25 s and last 0.75 s of the motion the diver maintains his body in fully extended position and during the rest of the time period of 1 s he maintains his body in tucked position. See figure 3. (Take $\pi = 3.0$). The diver rotates around OP at a rate of 0.5 revolutions per second during first 0.25 s.

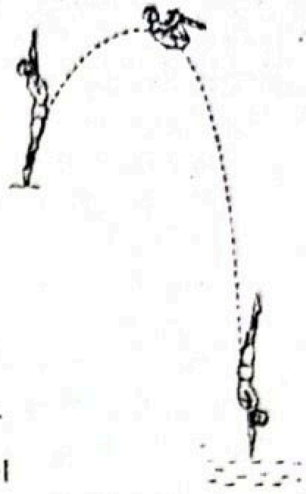


Figure 3

(i) Find the angular speed (ω_1) of the diver during first 0.25 s.

If the diver rotates $2\frac{1}{2}$ revolutions around OP during the total time period of 2 s find.

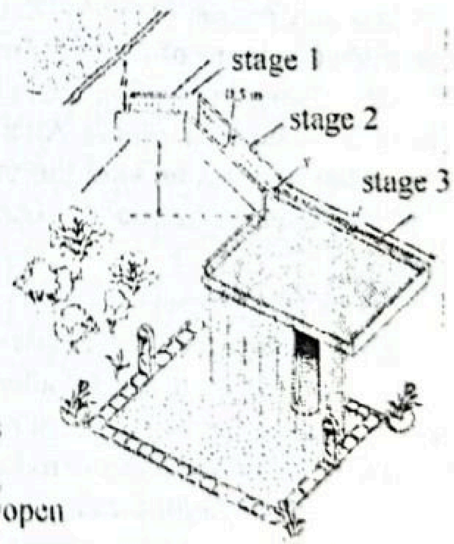
(ii) The angular speed (ω_2) when the diver is in fully tucked position.

(iii) The moment of inertia of the diver about OP in the fully tucked position. The moment of inertia of the diver about OP in fully extended position is 20 kg m^2

(iv) The rotational kinetic energy of the body when the diver is in fully extended position.

2009 A/L

20) Write down Bernoulli's equation and identify each term. An ancient waterway which supplies water to a pond consists of three stages as shown in figure.



Stage 1 – A rectangular horizontal open water channel originating from a rectangular outlet of a large reservoir as a depth h from the water level.

Stage 2 – Another rectangular open water channel having same floor, width as in stage 1 but runs with a slope as shown in figure. Width of the channel floor in stage 1 and 2 is 0.5m.

Stage 3 – The stage 3 linked to the stage 2 is an open horizontal shallow channel of rectangular cross – section with much broader floor width, d of 10 m. Water coming from stage 2 enters this channel and starts to flow in orthogonal direction as shown in figure creating a waterfall which provided water to the pond below.

a) As the steady state waterfall carries 1.5 m^3 of water per second. If the speed of water flow at the exit X of the stage 2 is 10 ms^{-1}, calculate the height of the water level of the channel of stage 2 at X.

b) Assuming that the height of the water level of the shallow channel of stage 3 is equal to the height of the water level of the stage 2 at X, calculate the speed with which water flows through the shallow channel.

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c) If the speed of the water flow at the horizontal channel of stage 1 is 5 ms^{-1}, calculate the height of the water level of the open channel of stage 1.

d) Considering a stream line along the top surface of the water flow, calculate the height (y) from channel floor of stage 2 at X to channel floor of stage 1 (see the figure). You may assume that the water leaves to the atmosphere of atmosphere pressure P at the outlet of the reservoir, and water enters the shallow channel at X which is also at pressure P.

e) Calculate the height h of the water level in the reservoir that has to be maintained for this purpose.

f) If the water level of the reservoir exceeds the value calculated in (e), propose a method to regulate water flow so that the waterfall carries the same amount of water per second, mentioned in (a)

2010 A/L

21) A uniform rod, AB of square cross section, mass M and length L rests on a horizontal frictionless surface as shown in figure 1. The moment of inertia of the rod about an axis perpendicular to the surface and passing though its centre of gravity, G is I.

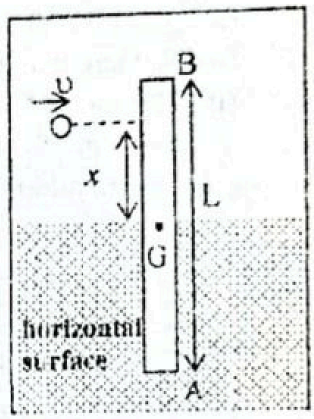

figure 1

The rod is struck by a ball of mass m travelling along the surface without spinning, with a velocity υ perpendicular to the rod. The motion of the rod due to the impact of the ball can be studied in terms of the linear motion of the centre of gravity of the rod and the rotation about its centre of gravity. Assume that the rod does not topple. After the impact the ball recoils in the opposite direction with the same speed. First consider the linear motion of the rod that occurs due to the impact of the ball.

(a) (i) Write down an expression for the linear momentum of the ball before the impact.

(ii) Considering only the linear motion of the rod, obtain an expression for the velocity V, of the rod after the impact.

(b) Now consider the rotational motion of the rod around its centre of gravity

(i) If the ball strikes the rod at a distance x from its centre of gravity, write down an expression for the angular momentum of the ball about the centre of gravity of the rod before the impact.

(ii) Considering only the rotational motion of the rod about its centre of gravity, obtain an expression for the angular velocity, ω of the rod about its centre of gravity after the impact.

(c) (i) Using the expression obtained in (b), (ii) above, write down an expression for the linear velocity, υ' of the end A of the rod due to the rotational motion of the rod.

(ii) Are the directions of V and υ' same or opposite ?

(iii) For a certain value x_s of x the end A of the rod remains at rest as the rod begins to move. Derive an expression x_s.

(d) The moment of inertia I, of the rod about its centre of gravity is given by $I = \frac{1}{12}ML^2$.

If $L = 0.6$ m, determine the value for x_s obtained in (c) (iii) above.

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(e) Consider a player holding a tennis racket at the end of its handle (see figure 2). When the ball is struck at the special point at distance x_S, from the centre of gravity of the racket, no force is produced on the palm of the player and it minimizes the 'string' the player experienced on the palm.
Indicate on your answer script, by drawing an arrow, the direction of the force on the palm experienced by the player when,

(i) $x > x_S$ (ii) $x < x_S$

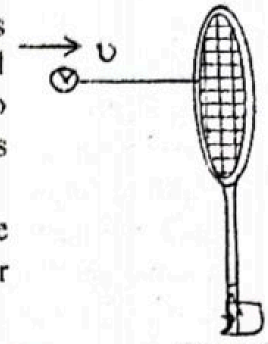
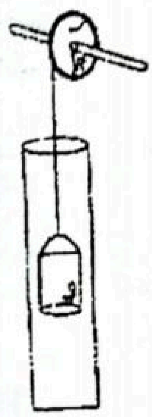


Figure 2

2011 A/L

22) A capsule which is free to move through a vertical cylindrical tube as shown in figure can be used to rescue a person trapped in an underground mine. A wire, one end of which is fixed to a pulley of radius R, and wrapped around the pulley, is used to hang the capsule. Assume that mass of the wire and the friction between the wire and the pulley are negligible. The pulley is free to rotate about a horizontal axle. Answers to the following questions should consist of only relevant quantities represented by the given symbols. (g = gravitational acceleration)



(a) For this part assume that the mass of the pulley and the frictional force against the rotational motion of the pulley are negligible.
- (i) If the capsule of total mass M is released from rest, use the law of conservation of energy to obtain an expression for the speed of the capsule after it has moved down a depth h.
- (ii) Find the angular speed of the pulley after the capsule has moved down the depth h.

(b) If the mass m of the pulley is not negligible and the moment of inertia of the pulley about the rotating axis $\frac{1}{2}mR^2$, repeat parts (a) (i) and (a) (ii) neglecting the frictional forces.

(c) Under practical situations the mass m of the pulley and the friction against the rotational motion are not negligible. Assume that the friction exerts a constant frictional torque, (τ_f) against the rotational motion of the pulley
- (i) What is the work done against the frictional torque (τ_f) when the pulley has rotated by an angle θ_0 in radians?
- (ii) Answer parts (a) (i) and (a) (ii) under these conditions.
- (iii) After moving down a depth h_0 the capsule reaches the bottom of the tube and stops. However the pulley keeps rotating against the frictional torque. Use the law of conservation of energy to find the number of turns (n) that the pulley would rotate further after the capsule has stopped.

(d) A person of mass m_0 gets into the capsule when it, is at the bottom of the table. Find the external torque (τ_e) that must be applied on the pulley to rotate it, at a constant angular speed while raising the capsule. Assume for conditions given in part (c) fort this.

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23) In this question, you will investigate a few basic movements of a robotic arm shown in figure (1)
The arm segments A and B of the robot have the ability to rotate in either direction around joints 1 and 2 in horizontal planes. Joint 3 allows segment C to move up and down. All three joints are operated by electric motors. Assume that only one movement around or across a joint is allowed at a given time and that there is no friction in any of the joints.

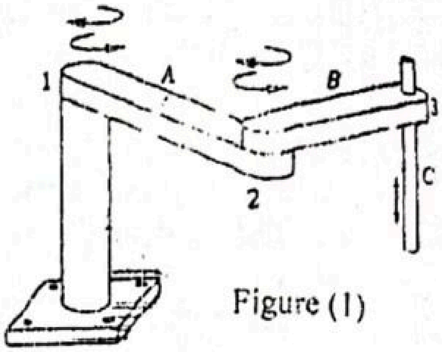

Figure (1)

a) First consider an upward motion of segment C. This motion is described by the velocity (υ) – time (t) graph in figure (2). Mass of segment C is 0.1 kg

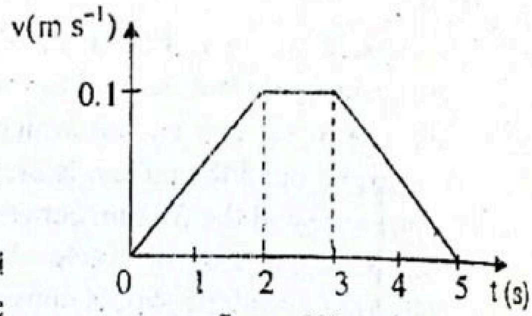

figure (2)

i) Calculate the acceleration of segment C during the first 2 seconds
ii) The forces acting on C are its weights, and the force applied by the motor for the motion of C. Calculate the force applied by the motor during the first 2 seconds
iii) What are the magnitude and direction of the force applied by the motor on C during the last 2 seconds of motion ?
iv) Suppose the magnitude of the maximum force the motor can exert on C is 1.2 N. If starting from rest. C moves up under this maximum force for 0.5s. how far will it move?

b) Next consider a rotation of segment B (together with segment C) occuring around joint 2. The angular velocity (ω) – time (t) graph in figure (3) shows this rotation. Assume that segment A is held fixed during this rotational motion.
The moment of inertia of the combined system of segments, B and C around the axis of joint 2 is 0.01 kgm^2.

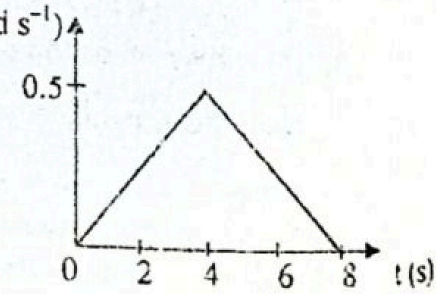

figure (3)

i) Calculate the torque applied by the motor on B during the first 4 seconds of motion shown in figure (3)
ii) Calculate the angular displacement of B during the 8s period shown in figure (3)
iii) If the magnitude of the maximum torque that can be applied by the motor is 0.002Nm, what is the **minimum** time that will take for B to start from rest and come to rest again after an angular displacement of 3.2 radians ?

c) Now if segment A is allowed to rotate freely around joint 1, what would be the direction of rotation of segment A, when segment B, starting from rest, rotates clockwise around joint 2? Give reasons for your answer.

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2013 A/L

24) The vertical force (lift) required for the taking off of an airplane is provided by two forces, one arises due to the Bernoulli effect and the other due to the hitting of air molecules on the wings of the airplane. The orientation and the cross sectional view of a wing of an airplane when it is travelling along the runway for taking off are shown in the figure (1). Here the bottom surface of the wing makes an angle θ with the horizontal direction.

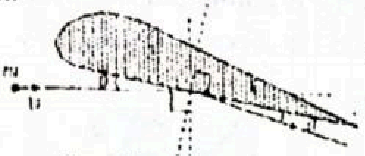

Figure (1)

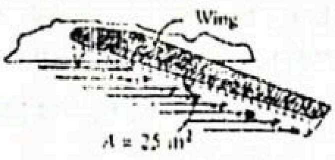

Figure (2)

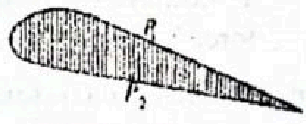

Figure (3)

a) Take the speed of the airplane on the runway to be v (ms^{-1}) at a certain instant and assume that air molecules remain still relative to the earth. Also assume that each air molecule has the same mass m. Consider a complete elastic collision of an air molecule with the wing. [see figure (1).] The speed of the air molecule relative to the airplane is shown in the figure.

   i) Write down an expression for the change in momentum of the air molecule in the direction perpendicular to the bottom surface of the wing in terms of m, v and θ.

   ii) If the number of air molecules hitting the wing during one second is N, using the result in (a) (i) above obtain an expression for the vertical force generated by the collisions of air molecules on the wing in terms of m, v, θ and N.

b) When the airplane is moving, a wing sweeps an effective cross-sectional area A [figure (2)], and therefore molecules in a volume Av hit the wing during one second period. Let the density of air be d.

   i) Write down the total mass of air molecules hitting the wing during one second in terms of A, v and d.

   ii) Hence express N in terms of A, v, d and m.

   iii) Obtain an expression for the total vertical force (take as F_c), generated due to the collisions of air molecules on both wings in terms of A, v, d and θ.

   iv) If $\theta = 10°$, $A = 25\ m^2$ and $d = 1.2\ kgm^{-3}$ obtain the value of F_c in terms of v. (Take $\sin\theta = 0.2$ and $\cos\theta = 1$ for $\theta = 10°$)

c) i) Assume that, because of the shape of the wing, the average speeds of the air streams relative to the airplane just above and just below the wing are $\frac{7v}{6}$ and $\frac{5v}{6}$ respectively. Taking P_1 to be the pressure just above the wing and P_2 to be the pressure just below the wing [figure (3)], show that the pressure difference across the wing due to the Bernoulli effect is given by $(P_2 - P_1) = \frac{2}{5} v^2$

   ii) If the effective surface area of one wing is 120 m^2, find the total vertical force on both wings (Say F_b) due to the above pressure difference, in terms of v.
   (Assume $\cos 10° = 1$)

d) If the mass of the airplane is 4.32×10^4 kg, calculate the minimum speed required for the plane to take off.

e) The maximum possible acceleration of the airplane on the runway is 0.9 ms^{-2}. Assuming that the airplane accelerates uniformly, calculate the length the runway must have for taking off.

f) Pilots take off airplanes by accelerating against the direction of wind, whenever possible. Explain the reason for this.

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2014 A/L

25) a) When a person is changing steps while walking, at one instant the entire body weight of the person is borne only by a single leg as shown in figure (1). Front view of the relevant bone structure of this leg is shown in figure (2), and the corresponding simplified free-body diagram indicating all the forces acting on the leg is shown in figure (3). All the forces indicated in figure (3) and the weight of the body are acting in one vertical plane and the frictional force between the leg and the ground is negligible for this situation.

Figure (1)

Here, F_M = Resultant force acting on the leg by the group of muscles M

F_S = Force exerted by the hip socket (S) on the leg

W_L = Weight of the leg

R = Reaction force acting on the leg by the ground

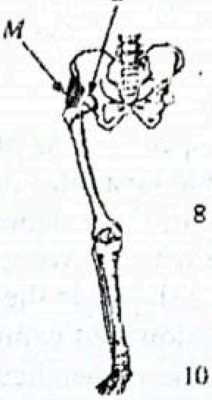

Figure (2)

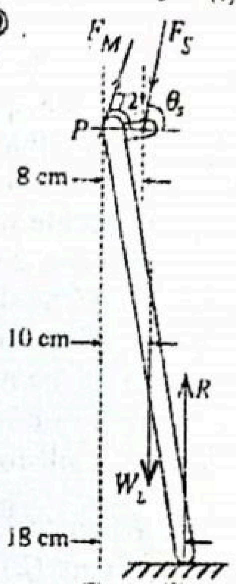

Figure (3)

(i) If the weight of the person is W, express the reaction force R in terms of W.

(ii) Generally, $W_L = 0.2W$. By taking the moments around the point P or otherwise obtain a relationship between F_S, θ_S and W.

(iii) Find F_M in terms of W. (Take $\sin 72° = 0.9$ and $\cos 72° = 0.3$)

(iv) Find the value of θ_S.

(v) Find F_S in terms of W. (Only for this calculation, you may take $\sin \theta_S = 1$)

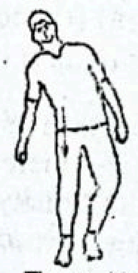

Figure (4)

b) When a person with an injured hip joint is walking, he tends to limp by leaning towards the injured side as he steps on the foot attached to the injured joint [see figure (4)]. As a result, the centre of gravity of the body shifts to the side of injured hip joint and F_M acts in the vertical upward direction. The free body diagram for the leg for this case is shown in figure (5) and corresponding forces of F_M and F_S are indicated as F'_M and F'_S respectively.

(i) Find the force F'_S for this situation, in terms of W.

(ii) Calculate, as a percentage, the reduction in the magnitude of force F_S as a result of the limping of the person for the reason described in (b) above.

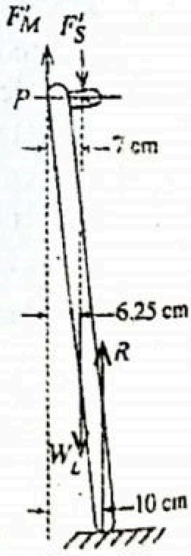

Figure (5)

c) In the process of walking, while one leg rests on the ground the other leg moves around the hip joint. This motion can be considered as an oscillatory motion of a rod which is freely pivoted at one end as shown in figure (6). Here the leg is considered as a uniform rod of length l.

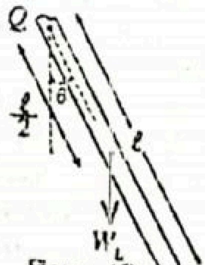

Figure (6)

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(i) If I is the moment of inertia of the rod around the axis of rotation through the point Q, obtain an expression for the angular acceleration α in terms of l, θ, W_L, and I, for the position indicated in figure (6).

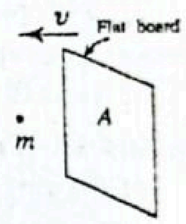
(ii) The period of oscillations of the rod, T, can be obtained from $T = 2\pi\sqrt{\frac{\theta}{\alpha}}$ and it can be shown that $T = 2\pi\sqrt{\frac{2l}{3g}}$ for a uniform rod of length l.

Calculate the value of T corresponding to a person whose length of a leg is 0.9 m. Take $\pi = 3$ and $\sqrt{0.06} = 0.25$

(iii) The most effortless speed of walking for a person is the speed where his legs oscillate with the period obtained in (c) (ii) above. When a person with 0.9 m long legs is walking, the distance between two successive positions where one of his legs touches the ground is 0.9 m. Calculate the most effortless walking speed for him.

2015 A/L

26) a) A vertical flat board of cross-sectional area A moves in still air at a constant speed v as shown in the figure. Consider the relative motion of the board and the air molecules. Under this condition, assume that the air molecules collide with the surface of the board perpendicularly and after colliding, bounce back in the opposite direction with the same speed v with respect to the board.



i) If m is the mass of an air molecule, write down an expression for the change in momentum of the molecule.

ii) Considering the number of air molecules colliding with the board per unit time or otherwise, show that the magnitude of the force F exerted on the board by the air can be given by $F = 2Adv^2$, where d is the density of air. **This force is known as the drag force.**

b) The drag force (F_D) on an object moving in a fluid depends on the shape of the object. A more accurate expression for F_D can be given as $F_D = KAdv^2$, where K is a constant which depends on the shape of the object. Drag force plays an important role in the design of the external shape of vehicles.

Consider a motor vehicle moving in still air on a flat road with a constant speed v. Take $K = 0.20$, $A = 2.0\ m^2$ for the motor vehicle and $d = 1.3\ kgm^{-3}$.

i) Write down an expression for the power (P) needed to overcome the drag force F_D.

ii) Calculate the power P when the motor vehicle is moving with a speed of 90 km h^{-1} (=25 ms^{-1})

iii) If the power needed to overcome other external frictional forces acting on the motor vehicle in order to maintain a constant speed of 90 km h^{-1}?

i) If the speed of the motor vehicle has been increased from 90 km h^{-1} to 126 km h^{-1} (= 35 ms^{-1}), calculate the additional power required to maintain the speed of the motor vehicle at that value.

ii) If the motor vehicle climbs at a constant speed of 90 km h^{-1} on a road of slope of 3°, calculate the additional power that should be supplied by the drive wheels. Consider that the mass of the motor vehicle is 1200 kg. (Take $\sin 3° = 0.05$)

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c) Consider a motor vehicle moving on a flat road as described in b(iii) above. Consider that the energy released by burning one litre of petrol is 4 x 107 J and only 15% of this energy is used to drive the wheels. Under following conditions, calculate the fuel efficiency of this motor vehicle in kilometres per litre.

i) When it moves in still air.

ii) When it moves in opposite direction to a wind blowing at constant speed of 36 kmh^{-1}(=(10 ms^{-1})

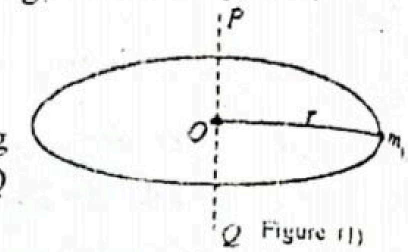

Figure (1)

2016 A/L

27) A particle of mass m_1 is fixed to the rim of a horizontal ring of radius r and negligible mass as shown in figure (1) POQ is a vertical axis passing through the centre O of the ring.

a) i) Write down an expression for the moment of inertia I_1 of the particle about the vertical axis POQ in terms of m_1 and r.

ii) Another particle of mass m_2 is now fixed to the rim of the ring which is diametrically opposite to m_1 and the system is rotated about the axis POQ with a constant angular speed ω. If I_2 is the moment of inertia of mass m_2 about the axis POQ. write down an expression for the total rotational kinetic energy (E) of the system.

iii) If I_0 represents the total moment of inertia of the above system in (a) (ii) about the axis POQ. using the expression obtained in (a) (ii) show that $I_0 = I_1 + I_2$.

b) Instead of m_1 and m_2, 10 identical particles, each of mass m, are now fixed to the rim of the ring with equal spacing. If I is the moment of inertia of a particle about the vertical axis POQ, Write down an expression for total moment of inertia (I_T) of the system about the vertical axis POQ.

c) Now the ring described in (b) above is fixed onto an axle of negligible moment of inertia and coinciding with the vertical axis POQ using symmetrically fixed spokes of negligible mass as shown in the figure (2). The system is then started rotating from rest at time $t = 0$ in a horizontal plane about the axis POQ with a constant angular acceleration α and reached a constant angular speed ω.

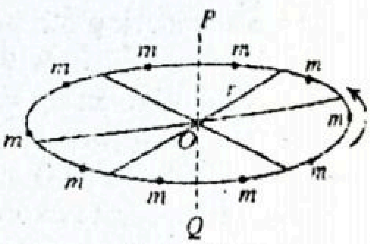

Figure (2)

i) 1) Obtain an expression for the time t taken by the system to reach the constant angular speed ω.

2) How many revolutions have been made by the system when it reaches the constant angular speed ω?

ii) Write down an expression for the centripetal force (F) acting on one particle when it is rotating about the axis POQ with a constant angular speed ω.

d) The structure of the merry-go-round shown in figure (3) which is at rest is similar to the structure of the system described in (c) above. However, instead of fixed mass m, the system has 10 chairs occupied by riders and hung by chains of negligible mass. The moment of inertia of the merry-go-round, **without riders and chairs**, about the axis POQ is 32 000 kg m^2.

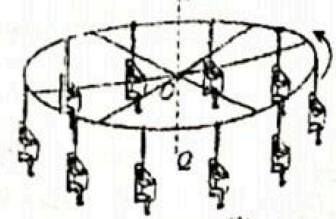

Figure (3)

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Consider a situation where the merry-go-round is rotating about the axis POQ with a constant angular speed of 12 revolutions per minute with all the chairs being occupied by riders. When the merry-go-round rotates, all the chains are inclined to the vertical by an angle θ, and figure (4) shows the situation with respect to one rider.

Use $\pi = 3$ for relevant calculations.

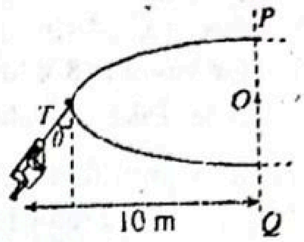


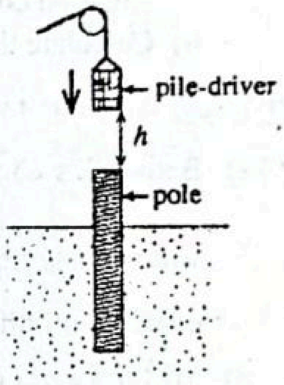
Figure (4)

i) If the riders are of mass 70 kg each and the chairs are of 20 kg each, calculate the total moment of inertia of the system about the axis POQ. When calculating the moment of inertia assume that the total mass of the rider and his chair is **concentrated** at a horizontal distance of 10 m from the axis POQ.

ii) Calculate the value of θ.

2017 A/L

28) The "pile-driver" is a heavy weight which is used to drive poles called piles into the ground for use as foundations of buildings and other structures. As shown in the figure (1), the pile-driver is lifted up by a cable and then dropped so that it falls freely under gravity and strikes the top of the pole. This process is repeated until the pole is pushed to the desired depth into the ground.



a) Consider a situation where a pile-driver of mass $M = 800\ kg$ is raised and then released from rest on to a vertical cylindrical pole of mass $m = 2400\ kg$ from a height $h = 5\ m$.

i) State the energy conversion that takes place during the fall of pile-driver

ii) Calculate the speed of the pile-driver just before the collision.

iii) Calculate the magnitude of the momentum of the pile-driver just before the collision.

b) Assume that after collision between the pile-driver and the top of the pole, the pile-driver does not bounce back, instead it remains in contact with the pole and drives the pole into the ground vertically. Also assume just the collision, only the momentum is conserved in the system. Calculate, the following.

i) The speed of the pile-driver with pole just after the collision.

ii) The kinetic energy of the pile-driver with pole just after the collision.

iii) In each collision 40% of the energy calculated in (*b*) (*ii*) is used usefully to drive the pole into the ground. If in one particular collision it drives the pole 0.2 *m* into the ground, calculate the average resistive force acting on the pole.

c) Consider a situation where a uniform cylindrical wooden pole of 10 *m* height and 0.3 *m* radius is pushed entirely into a sandy soil as shown in the figure (2). The maximum load F the pole can hold when keeping it as shown in figure (2) could be written as $F = A_s f_s + A_b f_b - W$,

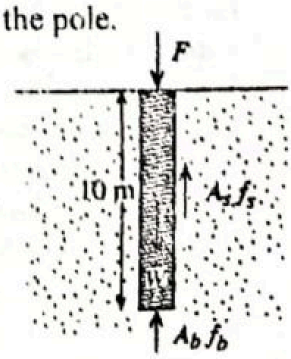


Figure (2)

where W is the weight of the pole, A_s is the area of the curved surface of the pole which is in contact with the

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soil. f_s is the average resistive force on the curved surface of the pole per unit area, A_b is the cross-sectional area of the base of the pole and f_b is the average resistive force from the ground on the base of the pole per unit area. If $f_s = -5 \times 10^4$ Nm^{-2}, $f_b = 2 \times 10^6$ Nm^{-2} and the density of the wood is 8×10^2 kgm^{-3}, calculate the value of F for the pole. Take the value of π as 3.

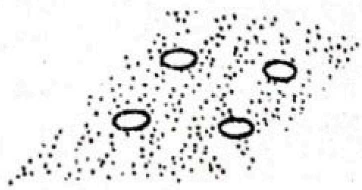
Figure (3)

d) System of four poles, each similar to the pole used in (c) but having a radius equal to half of the radius of the pole used in (c), is pushed entirely into a sandy soil. This is shown in the figure (3) when seen from the above.

i) As given in (c) above, the F has three components as A_sf_s, A_bf_b and W. When using the system of four poles for a construction, which component of the F, for the system of four poles, is contributing to increase its value in compared with the situation considered in (c) above.

ii) Calculate the value of F for the system of four poles.

2018 A/L

29) a) Bernoulli's equation for a fluid flow can be written as $P + \frac{1}{2}dv^2 + hdg = \text{constant}$, where all symbols have their usual meaning. Show that the term $\frac{1}{2}dv^2$ has the unit of energy per unit volume.

b) Sri Lanka has one of the most advanced ancient irrigation systems in the world. Such an irrigation system which supplies water for farmers and villagers consists of three major features as shown in figure (1).

Feature 1 : The tank or reservoir and the dam.

Feature 2 : The outgoing water canal from the tank which is exposed to atmosphere

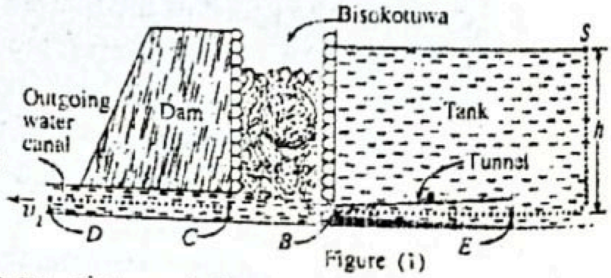

Figure (1)

Feature 3 : The Bisokotuwa (also known as cistern sluice) is a rectangular shaped vertical tower chamber with walls made of stones or bricks (see figure (1)). When it is required to release water from the tank, the water is first allowed to enter the Bisokotuwa in which the speed of the water flow is drastically reduced. One reason for this reduction is the sudden increase in the cross-sectional area of the water flow within the Bisokotuwa. In addition, a substantial amount of energy of the water flow is also lost, within the Bisokotuwa, due to the collision of water with the stone walls of the Bisokotuwa. For your calculations, assume that the steady and streamline flow conditions can be applied along the dotted line paths shown in figures and the height of the water level in the tank remains unchanged. Consider an irrigation system which consists of only the features 1 and 2 as shown in figure (2.)

i) If the height of the water level in the tank is h, derive an expression for the speed v_1 of the outgoing water at point Q in terms of h and g.

ii) If h = 12.8 m, calculate the value of v_1.

iii) Calculate the kinetic energy per unit volume carried by the water at point Q. The density of water is 1000 kgm^{-3}.

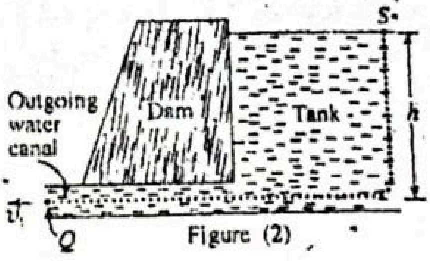

Figure (2)

c) To control the destructive power of the outgoing water, ancient engineers incorporated the feature 3, the Bisokotuwa to the tånk as shown in figure (1).

i) The water enters from the tank to the Bisokotuwa through a tunnel as shown in figure (1). Assume that the tunnel is tapered, and areas of cross-sections of the tunnel at the inlėt and outlet are A and 0.64 respectively. Calculate the speed v_B of the water flow at the point B in the tunnel. Take the speed of the water flow at the inlet E of the tunnel as 12 ms^{-1}.

ii) Calculate the pressure P_B of the water flow at the point B in the tunnel. The atmospheric pressure is 1×10^5 Nm^{-2}.

iii) Consider a point C in the outgoing water canal where the pressure and the speed of the water flow are at the values of 75% of P_B and 65% of v_B respectively.

(1) Write down the value of the pressure of water flow P_C at the point C.

(2) Write down the value of the speed of water flow v_C at the point C.

iv) Calculate the speed v_2 of the outgoing water at point D shown in figure (1).

v) Calculate the percentage loss, in kinetic energy per unit volume carried by the water at point D shown in figure (1) with respect to the value calculated in (b) (iii) above.

vi) Explain briefly, how ancient engineers managed to control the destructive power of the outgoing water flow by adding the Bisokotuwa to the irrigation system.

2019 A/L

30) a) In electric power generators, the frequency of the output voltage depends on the number of magnetic poles P and the number of revolutions per minute N of the generator. The frequency f in Hz is given by $f = \frac{P \times N}{120}$.

A portable generator consisting of two magnetic poles typically works at 3000 revolutions per minute (rpm).

Find the following:

(i) The frequency of the output voltage of the generator

(ii) The rotational speed of the generator in radians per second (rad s^{-1}) (take $\pi = 3$)

b) A student has designed a model of a hydro-power plant by replacing the engine of the portable generator mentioned in (a) above, with a turbine that can be rotated by a water flow. He observed that the frequency of the output voltage varies with the consumption of electricity even at a constant water flow. To control the frequency variation of the output, he designed a controlling device to adjust water flow to the turbine. Schematic diagram of the controlling device connected to a throttle valve is shown in figure (1).

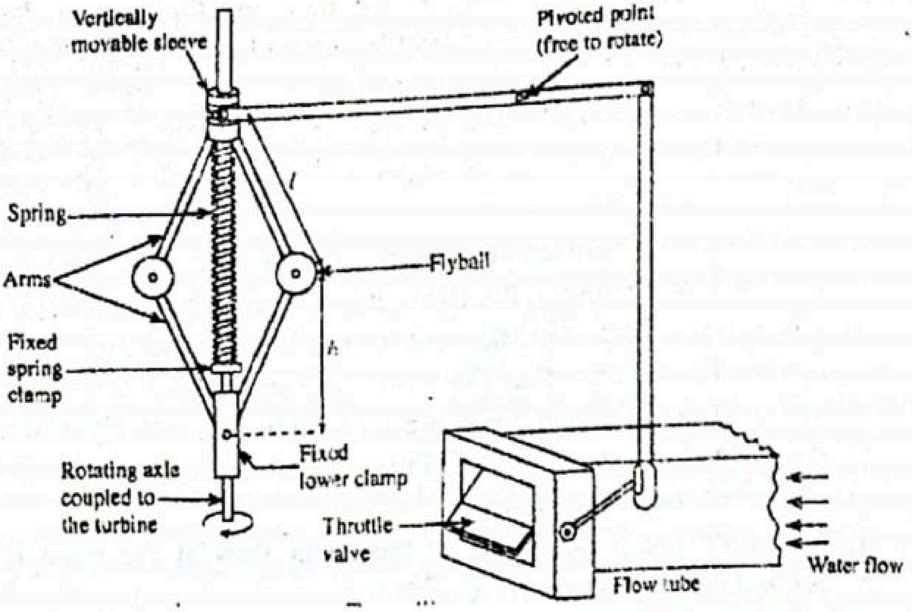

Figure (1)

Assume all the joints in this device are free to move without friction. During the rotation, flyballs move horizontally and it makes the sleeve move up and down along the rotating axle. This device is symmetric about the rotating axle. Opening and closing of the throttle valve is automatically controlled by the rotational speed of the turbine. All the other parts of the device can be assumed to be massless except the flyballs.

(i) Draw the free body force diagram for a flyball assuming each arm connected to it, is under tension. Consider the mass of a flyball to be m.

(ii) If the angular velocity of each flyball about the rotational axle is ω rad s^{-1}, show that the tensions in the upper and lower arms are respectively given by

$\frac{ml}{2}\left(\omega^2 + \frac{g}{h}\right)$ and $\frac{ml}{2}\left(\omega^2 - \frac{g}{h}\right)$ Here l is the length of each arm and h is the height to each flyball from the lower clamp.

(iii) When the frequency of the output voltage is 50 Hz, the value of h is 30 cm. Show that the contribution to the tension from the tern $\frac{g}{h}$ can be neglected.

(iv) If m = 1 kg and l = 50 cm, calculate the tension in an upper arm.

(v) When the frequency of the output voltage is 50 Hz, the contraction of the spring is 20 cm. Determine the spring constant of the spring.

(c) When the frequency of the output voltage is 50 Hz, the throttle valve is set to block 50% of the flow, That is, the valve is making an angle of 45° with the axis of the flow tube as shown in figure (2). Assume that the closing of the throttle valve is proportional to the angle of the valve with the axis of the tube.

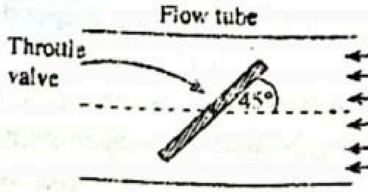

Figure (2)

The frequency of the output voltage depends on the consumption of electricity. When the consumption , increases, the output frequency decreases and vice versa.

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i) According to the design, when the frequency of the output voltage becomes 25 Hz, the throttle valve will be fully opened. The valve will remain fully open even for frequencies lower than 25 Hz. Determine the following at the instant of fully opening the throttle valve. (Neglect the contribution from the term $\frac{g}{h}$)

(1) Tension of an upper arm

(2) Contraction of the spring

(ii) When the frequency of the output voltage increases, the throttle valve closes gradually to decrease the flow rate. If the flow is to be blocked by 75%, what should be the frequency of the output voltage?

2020 A/L

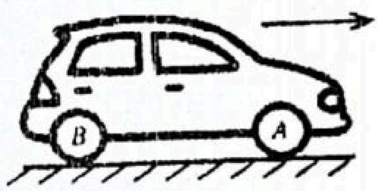
31) a) A uniform block of mass M is initially at rest on a horizontal rough plane. Then a horizontal force (P) which is gradually increased from zero is applied to the block. Let the frictional force be F.

i) Draw a free body diagram of the block for the above situation and name all the forces.

ii) Sketch the graph of F against P from the initial position until the block moves with acceleration. Mark the limiting frictional force (F_l) and the dynamic frictional force (F_D) on the graph.

iii) Write down expressions for the coefficient of limiting friction, u_L, and the coefficient of dynamic friction u_D.

b) In front – wheel drive cars, the engine is coupled to the front wheels through axles to drive the car. Consider a front – wheel drive car moving on a horizontal straight rough tar road as shown in the figure. The coefficients of friction between the tyres and the tar road are $u_L = 0.8$ and $u_D = 0.5$ respectively. Consider only the limiting or dynamic frictional forces acting on the car when solving problem problems below unless otherwise stated.



i) The situation when the car accelerates on a horizontal straight rough road is shown in the diagram. Copy wheels A and B of the diagram into your answer sheet and mark the frictional force on a front – wheel (A) as F_A and on a rear – wheel (B) as F_B. Also compare the magnitudes of F_A and F_B when accelerating.

ii) The mass of the car including the driver is 1200 kg which is equally distributed other all the four wheels. Identifying the correct coefficient of friction acting in this situation. calculate the maximum initial driving force of the car on the horizontal straight tar road.

iii) When the car is moving at uniform velocity of 72 kmh-1 on the horizontal straight road, the total resistance against the motion is 520 N. Find the power of the car at that velocity.

iv) Next the car climbs a steep road with angle of inclination 12^0 to the horizontal at the same power as in (b) (iii) above. If the total resistance against motion is now 200 N. find the maximum velocity of climbing. Use $\sin(12^0) = 0.2$.

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v) i) While the car was again moving at uniform velocity of 72 kmh^{-1} on the horizontal straight road, the driver suddenly saw an obstacle on the road at a distance 35 m. When he quickly applied brakes, all four wheels were locked and the tyres started to slip without rolling identifying the correct coefficient of friction acting in this situation and by giving appropriate reasoning and calculation. State whether the arc would hit the obstacle or not. Neglect the reaction time of the driver before braking.

ii) If tyres are slipping when brakes are applied, then the car will move longer distance in a straight line without control which can cause accidents. To avoid such slipping of tyres without rolling, cars are equipped with an anti-lock braking system (ABS). When tyres start to slip during braking. ABS automatically releases the brakes and allows tyres to roll again. This process happens several times a second and the effective coefficient of friction brings close to the balue that of limiting friction. When the car is fitted with an ABS, the effective coefficient of friction becomes 0.75. Calculate the new stopping distance of the ABS fitted car for the same situation mentioned in (b) (v) (I) above.

vi) Then the car enters a horizontal circular road of radius of curvature 18 m. Assuming that the coefficients of friction are same as in part (b) above, find the maximum velocity that the car can move safely without slipping.

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