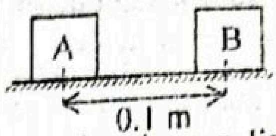
Gravitational field

1991 A/L

1) State Newton's law of universal gravitation.



Two identical blocks A and B of mass 100 kg each are placed on a light rough horizontal surface in a region where there are no other objects except the earth which exerts an attractive force on A and B in the vertically downward direction. The centres of gravity of the blocks are 0.1 m apart as shown in the figure and the coefficient of static friction of the horizontal surface is 0.1 .

i) Taking the universal gravitational constant (G) to be equal to 6.67×10^{-11} $Nm^2 kg^{-2}$. find the magnitude of the gravitational force acting on B due to A. Indicate the direction of this force on B.

ii) does the block B move towards A due to the gravitational force mentioned in (i)? Explain your answer.

iii) Is there a frictional force acting on B? If so, what is its magnitude?

iv) Calculate the minimum mass that the block A should have in order to set the block B.

1998 A/L

2) Write down Newton's law of gravitation in the form of an expression, identifying the symbols used. Obtain an expression for the acceleration due to gravity (g) at the surface of the earth in terms of mass (M) and radius (R) of the earth.

A satellite of 1000 kg is placed in a circular orbit so that it goes round the earth 10 times a day. Radius of the earth is 6.4×10^6m.

(i) Find the height of its orbit from the surface of the earth.

(ii) Calculate the total energy of the satellite in its orbit

(iii) Find the minimum energy required to take the satellite from the surface of the earth to its orbit.

(iv) Explain why the values obtained for (iii) and (ii) are different.

(v) At what height above the earth surface, the satellite must orbit to be a geostationary satellite ?

(vi) If a satellite in orbit loses energy due to friction what happens to the speed of the satellite and the radius of the orbit.

2003 A/L

3) A space lab is in a circular orbit of 1700 km above the earth surface.

(i) What is the speed of the space lab? Radius of the earth is 6400 Km and gravitational acceleration (g) on the earth surface = 10 m s^{-2}

(ii) Calculate the minimum energy that has to be supplied to a space vehicle of mass 10^4 kg including cargo to just reach the orbit of the space lab from the earth. Neglect air resistance.

(iii) What additional energy is required for the space vehicle to link with the space lab without changing its orbit?

(iv) After the linkage, the cargo in the space vehicle is transferred to the space lab. Will the speed of the orbiting space lab change due to loading this cargo? Explain your answer.

2006 A/L

4) Consider a hypothetical situation in which a point object of mass m carrying a charge $+q$ is fixed at a point in space where there are no other objects and charges present except at infinity.

(i) Show that if $m = \frac{q}{2\sqrt{\pi G\varepsilon_0}}$, a second object having identical mass and charge can be brought from infinity towards the first object without doing any work. (Neglect the energy required to initiate the motion of the second object at infinity.) G is the universal gravitational constant ε_0 is the permittivity of free space. Show also that when bringing the second object from infinity,

(a) If $m > \frac{q}{2\sqrt{\pi G\varepsilon_0}}$, work is done by the object and

(b) If $m < \frac{q}{2\sqrt{\pi G\varepsilon_0}}$, work has to be done on the object.

a) What is the total work that has to be done when bringing the second object from infinity to a point at a distance r from the first object under the condition stated in (i) (b) above?

b) State under which condition given in (i) above, the second object has a capability of existing in a circular orbit around the first object.

c) If the second object moves in a circular orbit of radius r as stated in (iii) above with a speed v_0 write down an expression relating r and other quantities mentioned above.

(v) An asteroid of mass m located far away from a certain planet of mass M starts to move towards the planet due to the gravitational influence between them. Assume that the planet is stationary and there is no gravitational influence on the planet and the asteroid from other celestial objects. If the speed of the asteroid when it is at a distance R away from the planet is v, derive and expression for the magnitude of the charge that has to be placed on each of the objects at that instant (i.e. at the instant when the separation is R) in order to stop the asteroid at a distance R/2 from the planet and reverse its motion.

2008 A/L

5) (a) If the mass and the radius of the earth are M and R respectively, write down an expression for the gravitational potential at a point P, which is at a distance h $(h > R)$ from the centre of the earth, in terms of M, h and the universal gravitational constant G. Assume that the gravitational potential is zero at an infinite distance from the centre of the earth.

(b) Suppose a small object of mass m is projected vertically upward from the point P with speed υ_1.

(i) Write down an expression for the total mechanical energy of the object at its starting point?

(ii) Obtain an expression for the maximum height H the object travels from the center of the earth, in terms of h, G, M and υ_1

(iii) Find an expression for the escape velocity υ_e' of the object in this situation, in terms of G, M and h.

(c) If υ_0 is the speed required to keep the object in a circular orbit at a distance h from the centre of the earth, show that $\upsilon_e' = \sqrt{2\upsilon_0}$.

(d) If, $M = 6 \times 10^{24}$ kg and $R = 6400$ km, calculate the escape velocity υ_e at the surface of the earth. Take $G = 6 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$ and $\sqrt{2} = 1.4$.

(e) The mean temperature of the surface of the earth is 280 K. Calculate the root mean square speed (υ_{rms}) for H_2 and O_2 molecules at this temperature.

Use following data for your calculations:

Boltzmann constant $= k = 1.4 \times 10^{-23}\ J\ K^{-1}$

Mass of a H_2 molecule $= m_{H2} = 3 \times 10^{-27}$ kg

Mass of a O_2 molecule $= mO_2 = 16 \times m_{H2}$

(f) For a given temperature gas molecules have a range of speeds from very fast to very slow. To remain a given gas in atmosphere the condition. $6\upsilon_{rms} < \upsilon_e$, must be satisfied for that particular gas. Using the results obtained in (*e*) above, explain why oxygen gas exists in the earth's atmosphere but not hydrogen gas.

2011 A/L

6) Use of satellite is expanding due to many application in areas such as communication, meterology, defence and scientific exploration about the earth as well as the outer space. Satellite are placed on certain orbits depending on their applications. The gravitational force provides the required centripetal force to maintain a satellite in an orbit.

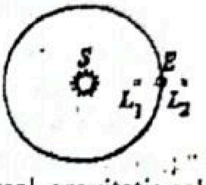
Geosynchronous satellites orbit the earth with a period of 24 hours, thus matching the period of the earth's rotational motion. A **Geostationary satellite** (GSS) is a Geosynchronous satellite in an approximately circular orbit on the plane passing through the earth's equator (0° latitude) that appears motionless in the sky, to a ground observer. The idea of a GSS was first proposed by the science friction writer Arthur C. Clarke. Communication satellite and weather satellites are oftern given Geostationary orbits as they can continuously observe the same areas on the earth. GSS used directional antennas for communication with ground stations. There are also several disadvantages of a satellite being operated as a GSS. The number of satellites that can be maintained in Geostationary orbits without inserfering one another is limited. An electromagnetic (EM) signal emitted from a ground station, travels at the speed of light ($3 \times 10^8\ ms^{-1}$). Due to the great distance in the satellite a significant time delay is introduced between the original signal emitted from an earth station and the signal received by another station after travelling via a satellite. Furthermore, due to the greater height, the clarity of pictures of the earth taken by GSS are poor, especially at locations away from the equator. Another problem would be the damage caused by the EM radiation from the sun when a GSS comes lower to the sun especially when the sun passes through the equatorial plane at late March and late September.

Low **Earth Orbit Satellites (LEOS)**, typically operating at the heights of 160 – 2000km from the **surface of the earth** with shorter periods, have become popular in recent years. Their orbits could be on any plane passing through the center of the earth. However, for continuous data acquisition pertaining to a specific location (eg : observation of weather over a given country) a system of a group of LEOS is needed. Some advantages of a LEOS are the use of simple non – directional antenna, reduced time delay for EM signals, higher clarity pictures of the earth and less EM radiation from the sun. Also, it needs less energy and resources to place a satellite into a Low Earth Orbit and need less powerful amplifiers for successful communication. A polar satellite which passes over the poles of the earth is a special case of LEOS. Hubble space telescope is another example of LEOS.

For scientific exploration of outer space, experiments are conducted in observatories placed on the orbits which are far away from the earth. There are four specific locations called language points or L – points where satellites could be placed to perform such experiments. Satellites placed at L – points appear stationary relative to the Sun – Earth system. The following figure shows two of the L – points called L_1 and L_2. When the earth orbits the sun with a period of 1 year, satellite placed at L_1 and L_2 also move with the Sun – Earth system but the relative locations of them remain the same. There are four satellites at the vicinity of L_1 and three satellites including the latest Planck Space Observatory have been placed at the vicinity of L_2. L_2 is especially useful for observation of outer space because, the earth partially blocks solar radiation falling towards the satellite at L_2 throughout the motion. (Radius of the earth is 6.4×10^6m)

(a) What is the value of the period of a GSS ?

(b) Draw a 3 – dimensional diagram of the orbit of a GSS around the earth. Clearly indicate the geometrical North, South and the equatorial plane of the earth.



(c) Give an for a LEOS

(d) Obtain an expression for the radius r of a GSS in terms of universal gravitational constant G, mass of the earth M_E and the period T of GSS. Substitute the correct numerical values in the expression $GM_E = 40 \times 10^{13}\ m^3\ s^{-2}$. No need simplify the answer.

(e) Calculate the time delay in receiving an electromagnetic test signal emitted from a ground station to a GSS located 36000 km vertically above it, if the signal is received again by the same station.

(f) International space station orbiting around the earth is in an orbit with a radius of 6700 km inclined to the equatorial plane. Calculate its period. Is this a GSS or LEOS? Give the reason for your answer. ($\sqrt{67^3} = 67^{\frac{3}{2}} = 548.4$; ; Take π^2 as 10)

(g) Give three advantages of LEOS.

(h) Why is the location L_2 better for placing an outer space observatory ?

(i) Calculate the angular speed (ω) of the Plank Space Observatory in units of rad $year^{-1}$

(j) Write down an equation for the orbital motion of the Plank Observatory in terms of mass of the sun (M_S) mass of the earth (M_E) distance from the earth to the sun (R) and to the satellite (r), ω and G .Neglect the effect of other planets and the moon.

(k) Periods of satellite around any objects, is general should increase with the distance form the center of the object. Satellite at L_1 and L_2 are at different distances from the sun but have equal periods. Explain the reason for this.

2013 A/L

7) Read the passage and answer the questions given below.

Comets are small astronomical objects typically moving in highly elliptical orbits around the Sun. [see figure (1).] Some orbits extend roughly one light – year beyond the planetary system. The main force acting on the comet is the gravitational attraction to the Sun.

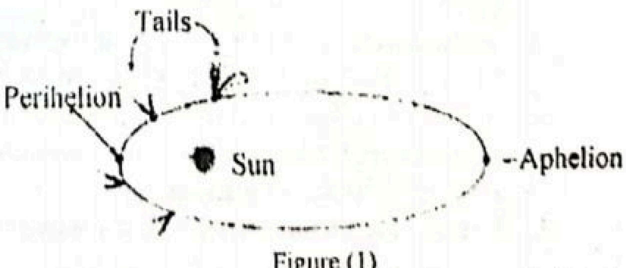


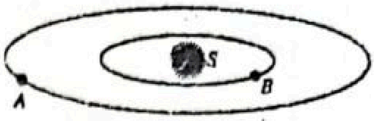
Figure (1)

The main components of a comet are the nucleus, coma and tails. While the solid body of the comet, the nucleus, is generally less than 50 km in extent, the coma may be larger than the Sun, and the tails can be extended over 150 million kilometres.
Comets mainly, compose of frozen carbon dioxide, methane, water (ice) with dust, and various types of minerals. When the comet reaches the inner planets and moving closer to the Sun, its outer layer gets evaporated due to the radiation pressure from the Sun. The dust and gases released from it form the extended atmosphere of the comet around its nucleus and is called the coma. The solar radiation pressure and the solar wind acting on the coma produce a bluish colour tail of ions which is straight and directed away from the Sun as the gas is more strongly affected by the solar wind. The dust released from the comet forms another white coloured and slightly curved tail behind the comet.
The speed of the comet varies from its minimum value at the furthest point from the Sun (aphelion) to the maximum value at the closest point to the Sun (perihelion). For example, the Halley's comet of mass 2.0×10^{14} kg at the aphelion which is at a distance of 5.0×10^{12} m from the Sun, acquires its lowest speed of 12.0 km s^{-1}.
Debris those enter the atmosphere from the outer space the known as meteoroids. Most meteoroids burn out emitting light in the atmosphere due to heat generated through the friction with the expense of their kinetic energies, both linear and rotational. They are called meteors. When the Earth's atmosphere crosses the debris left along the path of a comet, meteor showers could be observed. Some meteoroids fall onto the Earth's surface and they are called meteorites.
When a meteoroid is rapidly reaching its melting point, it becomes incandescent. When the surrounding atoms become ionized and rapidly recombined with the electrons causing the emission of light, the meteoroid produces a huge spherical air mass, appearing as a fire ball. Some meteoroids seen as fire balls could explode into several pieces of meteors. Seconds after seeing the explosion, the shock waves produced by the fragments of the meteoroid could reach the ground making ground – breaking sonic booms as heard in the recent event in Russia.

a) What are the main components of a comet?
b) Write down three main differences between the two types of tails of a comet.
c) Calculate the gravitational force acting on the Halley's comet when it is at aphelion.
   (Mass of the Sun = 2×10^{30} kg, $G = 6.7 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$)
d) Find the speed of the Halley's comet when it is located at the perihelion where its distance from the Sun is 8.0×10^{10} m.
   Note: The velocity of the comet is perpendicular to radial direction at both perihelion and aphelion. Assume that the mass remains unchanged.
e) Why meteor showers are produced when the Earth's atmosphere crosses an orbit of a comet?
f) What is the difference between meteors and meteorites?
g) What energies are converted to heat energy in burning meteoroids?
h) What is the mechanism that generates the light for a meteoroid to appear as a fire ball?
i) A meteoroid falling vertically downward with a speed of 200 m s^{-1} explodes into two pieces. If one piece having a mass 3/5 of the meteoroid travels in the horizontal direction with the speed of 600 m s^{-1}, find the speed of the other piece.
j) What should be the condition that must be satisfied by the speed of a piece of meteoroid to create a shock wave?
k) Explain the formation of a shock wave using a diagram.

2017 A/L

8) The main objective of NASA's Kepler exploration is to find habitable planets in other planetary systems in our galaxy, the Milky Way. A large number of planets which orbit around stars have been detected by the exploration. One such observation was a planetary system consisting of two planets, planet A and planet B of orbital periods $T_A = 300$ earth days and $T_B = 50$ earth days, respectively.



Assume that the planets are uniform spheres and moving in circular orbits around a star S of mass M as shown in the figure. Neglect the interaction between the planets.

a) i) Derive an expression for the orbital speed (v_B) of the planet B in terms of M, the orbital radius R_B of the planet B and universal gravitational constant G.

ii) Write down an expression for the period T_B of the planet B in terms of R_B and v_B.

iii) Derive an expression for the mass M of the star at the centre in terms of T_B, R_B and G.

iv) If $R_B = 0.3$ AU (1AU = 1.5×10^{11} m). Calculate the mass M of the star. Take $G = 6.7 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$ and $\pi^2 = 10$.

b) i) Using the expression obtained in (a) (iii) above, derive, an expression relating orbital radii R_A, R_B and periods T_A and T_B of planes A and B.

ii) Calculate the orbital radius R_A of planet A using given values.

c) The mass and the radius of the outer planet A are found to be 23 m_E and 4.6 r_E respectively, where m_E and r_E are the mass and the radius of the earth respectively.

i) Derive an expression for the gravitational acceleration, g_A, at a point on the surface of planet A, in terms of m_E, r_E and G.

ii) Obtain an expression for the g_A in terms of the gravitation acceleration g_E at a point on the surface of the earth.

iii) If a space landing module of mass 100 kg is landed on planet A, calculate the weight of the landing module after landing.

iv) The outer planet A is located within the habitable zone when compared with our solar system. Obtain an expression for the average for the average density d_A of the planet A in terms of the average density d_E of earth.

2020 (9) (A)

9) a) i) Write down an expression for the energy dissipation in a resistor of resistance R, when a direct current (d.c.) I flows through it in time t.

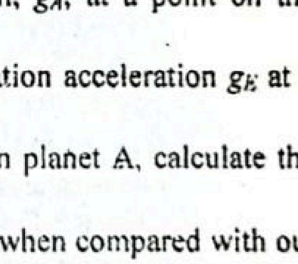


Figure (1)

ii) The variation of sinusoidal alternating voltage V with time t is shown in figure (1). Write down an expression for root mean square voltage V_{rms} in terms of peak voltage V_p.

iii) Out of the four lines A, B, C, D drawn in figure (1) which lines represent V_p and V_{rms} respectively?

iv) State the main advantage of using high tention a.c. voltage in long distance power transmission.

v) Rewrite the expression obtained for energy dissipation in (a) (i) above for a.c. currents.

b) A part of an electrical circuit connected to the a.c. main supply is shown in figure (2).

Following electrical appliances are connected to the main 230 V supply using a copper wire AB of cross sectional area 1 mm^2 and length 10 m. Assume that the voltage drop across AB is negligible.

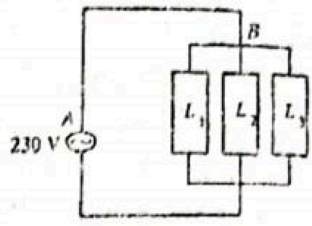


Figure (2)

L_1 - Rice cooker of 1200 W

L_2 - Refrigerator of 300 W,

L_3 - Electric kettle of 800 W.

i) Calculate the maximum current flow in the wire.

ii) Calculate the temperature rise when the maximum current flows through the wire for 10 s. Assume that the wire is completely insulated and no loss of heat to the outside. Mass of the wire is 100 g. Resistivity and the specific heat capacity of copper are 1.8×10^{8} m and 360 J $kg^{-1}\,{}^0C^{-1}$ respectively.

iii) Instead of a single copper wire a composite wire made of few such wires connected in parallel is used in high current flowing applications. Explain how this arrangement reduces hear dissipation.

c) An electricity meter measures the amount of electrical energy consumption in kWh. It uses eddy currents to rotate a thin circular aluminum disc. The number of revolutions of the aluminum disc is directly proportional to the electrical energy consumption.

i) A solenoid is placed other the horizontal aluminum disc, The number of revolutions of the aluminum disc, perpendicular to its plane as illustrated in figure (3). Suppose that the current through the solenoid is increasing in the direction as indicated in the figure.

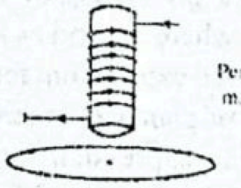


Figure (3)

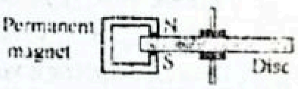


Figure (4)

Copy the figure (3) in to your answer sheet and draw the magnetic flux lines due to current in the solenoid and eddy current loops on the disc, indicating their directions.

ii) To decelerate the free revolutions of the disc when the power consumption is stopped, a permanent magnet is fixed as shown in the figure (4). Explain how the deceleration of the disc happen.

d) During the period from 6.00 p.m. to 10.00 p.m for a particular day at a house, the number of revolutions per minute (r.p.m) of the disc is measured. The graph in figure (5) shows its variation. The electricity meter is calibrated in such a way that 500 rotations is equivalent to 1 kWh.

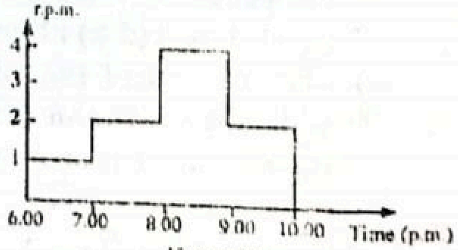


Figure (5)

i) Calculate the electrical power consumption at 8.30 p.m.

ii) If the electricity unit price between 7.00 p.m. to 9.00 p.m. is Rs. 40.00 per kWh and rest of the time is Rs. 10,00 per kWh, calculate the total cost for the period from 6.00 p.m. to 10.00 p.m.