Viscosity

1995 A/L

1) Using dimensional analysis show that the viscous force F acting on a sphere of radius a moving with a velocity V in a liquid of coefficient of viscosity η is given by

$$F = k\eta a V$$

Where k is a constant.

A sample of muddy water from a river is collected to a tall glass vessel and allowed to sediment at time t = 0. It can be assumed that the mud particles attain their terminal velocities within a negligible short time. Assume that the muddy water contains equal numbers of spherical particles of all sizes and initially they are distributed uniformly in the volume.

(i) Derive an expression for the terminal velocity V of a mud particle of radius a and density ρ moving down in water of density σ and coefficient of viscosity η

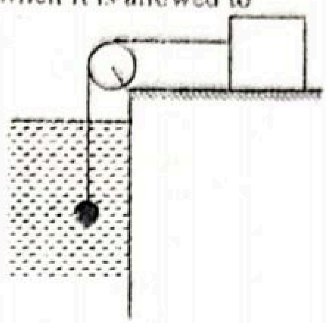
(ii) If $\rho = 2500\ kg\ m^{-3}$, $\sigma = 1000\ kg\ m^{-3}$, $\eta = 8 \times 10^{-4}\ N\ sm^{-2}$ and the height of water in the vessel is 1 m, calculate the time taken to sediment all the particles with $a = 8 \times 10^{-6}$ m. Assume that there are no collisions of particles inside the vessel.

(iii) Repeat the calculation in (ii) for particles having radius $a = 3 \times 10^{-6}$ m.

(iv) Once the sedimentation of particles having $a = 8 \times 10^{-6}$ m is over what fraction of particles with $a = 3 \times 10^{-6}$ m can be found inside that sediment layer.

1998 A/L

2) A sphere of radius 2×10^{-2} m attains the terminal velocity 3 ms^{-1} when it is allowed to fall from rest in a viscous liquid.



(i) Draw a rough sketch to show the variation of velocity (v) of the sphere with time (t)

(ii) When the sphere is connected to a block of mass 0.1 kg by a string passing over a pulley as shown in the figure, and allowed to fall in the same liquid, it attains the terminal velocity of 1.5 ms^{-1}. The coefficient of kinetic friction between the block and the horizontal surface on which it is placed is 0.4. The pulley is light and frictionless.

(a) What is the tension of string when the sphere attains the terminal velocity?

(b) Calculate the coefficient of viscosity of the liquid.

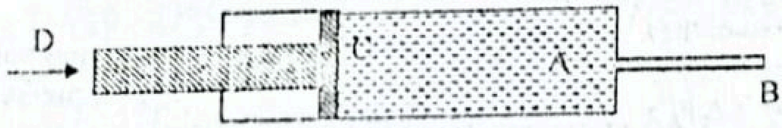
(c) Draw a rough sketch to show the variation of the velocity of the sphere with time on the same graph drawn in (i) above. Label the sketches properly.

(d) Calculate the new terminal velocity attained by the sphere if a layer of the same liquid of thickness 1 mm is present between the block and the surface. Contact surface area of the block 2.5×10^{-4} m^2

2000 A/L

3) Write down the Poiseuille's equation for a stream line flow of a viscous fluid through a tube, identifying all the symbols

Draw a labeled diagram of an experimental set up used in a school laboratory to find the coefficient of viscosity of water using the above equation



Briefly state the procedures that should be adopted to enhance the accuracy of the final result.

Figure shows a rough sketch of a syringe that can be used to inject liquid drugs into blood veins. The liquid inside the tube is slowly pushed out through the needle AB by applying a constant force on D.

Length of the needle is 2 cm and it had bore of radius 0.02 cm. Assume that the velocity of the liquid inside the wider tube is negligibly small under this situation.

(i) Draw the variation of the pressure inside the liquid along the axis of the syringe from C to B.

(ii) When the end B of the needle is exposed to the atmosphere the time taken to push 1 cm^3 of the liquid through the needle is 10 s. If the coefficient of viscosity of the liquid is 1×10^{-3} N s m^{-2}, calculate the pressure difference across AB.

(iii) Assuming that the average excess blood pressure over the atmospheric pressure is 100 mm of Hg, calculate the additional force to be applied on D in order to inject 1 cm^3 of the liquid into a blood vein in 10 s.

Density of Hg = 13.6×10^3 kgm^{-3}. Cross sectional area of the piston C = 0.75 cm^2

CS CamScanner

2001 A/L

4) A laminar flow of a viscous liquid is maintained on a stationary horizontal plate. The top layer of the liquid is moving with a constant velocity V and the stationary bottom layer is at a depth d.

(i) If the coefficient of viscosity of the liquid is η, write down and expression for the force F that has to be applied on the surface of an area A of the top layer.

(ii) Shown in a diagram using arrow, the variation of the velocities of the intermediate layers.

(iii) A person pushes a block of mass 0.5 kg on a horizontal floor. When a horizontal force 0.25 N is applied on the block it attains a constant velocity of $0.01\ ms^{-1}$. If a thin layer of an oil is applied on the floor, the horizontal force required to push the block with the same velocity $0.01\ ms^{-1}$ reduced to 0.05 N. The contact surface area of the block is $1 \times 10^{-2}\ m^2$ and the thickness of the oil layer is 1 mm.

(a) Calculate the coefficient of viscosity of the oil

(b) Find the effective coefficient of sliding friction between the block and the floor after applying the oil layer

(c) What is the energy that can be saved during one second due to the application of oil layer?

(d) In order to lift the block from the oil layered floor, in addition to the weight of the block a vertical force has to be applied upwards on the block. Explain the reason for this

2004 A/L

5) Write down poiseuille's equation for the flow of a viscous liquid through a tube, identifying the symbols.

(i) State two of the conditions under which posiculiie's equation is valid.

(ii) Suppose that the radius of cross section of the tube is r, the pressure difference across the tube is ΔP, and the volume rate of flow is Q.

(a) Write down an expression for the resultant force acting on the liquid in the tube due to this pressure difference ΔP.

(b) The average speed v of the liquid through the tube is given by $v = \frac{Q}{\pi r^2}$ show that this equation is dimensionaliy correct

(c) Hence show that the rate of work done by the pressure difference against the viscous force is $Q\Delta P$.

(iii) Poiscuille's equation is often used for approximate calculations of blood flow in human body:

(a) State two reasons why poiseuille's equation is not strictly valid for blood flow through vessels in human body

(b) If the average rate of blood flow through a horizontally situated artery with uniform cross section having radius 2 mm, and length 20 cm is $2.5\ cm^3s^{-1}$, calculate the pressure difference between its two ends. (Average viscosity of blood at body temperature is $4 \times 10^{-3}\ Nsm^{-2}$)

(c) Suppose the radius of cross section of the above artery has decreased to half the original value due to fat deposition.

(1) By how many times will the pressure difference across the artery have to be increased in order to maintain the same rate of blood flow mentioned in (iii) (b) above?

(2) By how many times will the rate of work done by the heart against viscous force have to be increased to maintain the same rate of blood flow as mentioned in (c) (1)

(d) sometimes doctors prescribe drugs that reduce blood viscosity, to patients having high blood pressure. Briefly explain how such drugs bring relief to patients.

2008 A/L

6) Poiseuille's equation can be written as $Q = \frac{\pi \Delta P r^4}{8\eta l}$.

Identify each physical quantity in the above equation.

(a) Show that above equation is dimensionally correct.

(b) Crude oil of viscosity $0.9\ kg\ m^{-1}\ s^{-1}$ and density $9.0 \times 10^2\ kg\ m^{-3}$ has to be delivered from a harbour to a refinery, using a straight horizontal metal tube of internal radius 20 cm and length 1 km, at an average speed of $1.0\ ms^{-1}$.

(i) Calculate the pressure difference that should be maintained across the tube.

(ii) What is the minimum power needed to deliver the oil through the pipe at the given rate? (Take $\pi = 3.0$)

(iii) At the radial distance the speed of oil in the tube has its maximum and minimum values? What is the value of the minimum speed?

(c) The internal radius of the metal tube is decreased by 10% due to deposits. What percentage the pressure difference across the tube should be increased in order to deliver oil at the same rate mentioned in (b) above ? (Take $\frac{10}{9} = 1.11$)

(d) Two smaller tubes having similar radii and lengths are now fitted to the end of the metal tube mentioned in (*b*) above, and oil is delivered to two other refineries, instead of the refinery mentioned in (*b*). If the length of a smaller tube is also 1 km, and the pressure difference across all the tubes are equal, find the radius of a smaller tube.

2010 A/L

7) a) Radii of water droplets in low – lying rain clouds is in the range of 10 μm to 60 μm. Under certain conditions smaller water droplets coalesce to form larger water drops and these water drops are releases from clouds as rain. How many water droplets, each having a radius of 10 μm, must coalesce to form a water droplet of radius 40 μm?

b) When a water drop is falling through air, a drag force is acting on the drop, in addition to the other two forces, weight and the upthrust. Only if the radius of the water droplet is less than 50 μm, the water droplet retains its spherical shape and the drag force is due to the viscosity of air which is given by Stokes' law. Consider a water droplet of radius 40 μm, released from a rain cloud located at an altitude of 2 km.

i) Assuming that the air remains at rest and the upthurst on the water droplet can be neglected, calculate the terminal velocity (υ_l) of the water droplet of radius 40 μm.
(Viscosity of air = 1.6×10^{-5} Pa s. Density of water = $\rho_w = 10^3$ kg m^{-3})

ii) It has been found that in general, a water droplet of radius 40 μm would evaporate completely within a period of 600 s, As the radius of this droplet decreases due to evaporation, the terminal velocity also decreases, and the mean velocity of the droplet for its entire motion can be considered to be $\frac{\upsilon_l}{2}$. Show that this water droplet will evaporated completely before reaching eh ground.

c) When the radius of the raindrop is larger (greater than about 100 μm) the shape of the raindrop tends to deviate appreciably from spherical shape. Now, consider the raindrop which has a vertical length h(>100 μm) and is falling vertically through air at **a constant speed**. Assume that the atmospheric pressure (Π) and the density of air remain constant. Take the radius of curvature of the drop as R_1 at the upper end of the drop and R_2 at the lower end.

i) If the pressure at a point just below the upper end of the water drop is, P_1 (> Π). write down an expression for ($P_1 - \Pi$) in terms of R_1 and the surface tension of water (γ).

ii) What is the pressure at a point just above the lower end of the raindrop ? Express your answer in terms of P_1, h the density of water (ρ_w) and acceleration due to gravity g.

iii) Show that $R_1 > R_2$.

iv) Calculate the value of ($R_1 - R_2$) for a raindrop of vertical length h = 4 mm. Take $R_1R_2 = 4\ 10^{-6}\ m^2$ for this case. Surface tension of water is $7.5 \times 10^{-2}\ Nm^{-1}$

d) When the maximum hydrostatic pressure inside the raindrop becomes greater than the pressure difference due to surface tension as the lower surface of the raindrop, the raindrop becomes unstable and breaks into smaller droplets. Assuming h = $2R_2$ calculate the maximum value of vertical length h_{max}, a raindrop can have. Take $\sqrt{7.5} = 2.7$.

2013 A/L

8) Treatment procedures adapted in hospitals very often require infusion of fluids such as saline, antibiotics, insulin, etc. into the venous system of patients, over a long period of time. A common method used for this is to allow the fluid to be infused to the patient under the gravity. Here, the fluid to be infused is included in a bottle, and a metal needle in the form of a thin tube is connected to the bottle by a plastic tube as shown in figure (1). The fluid is allowed to be infused by inserting the needle to a vein of the patient.

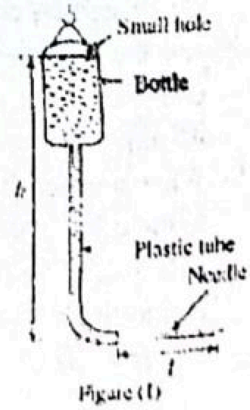


Figure (1)

a) Suppose that it is required to infuse a saline solution to a patient using the set – up shown in figure (1)

i) If r = Internal radius of the needle : l = Length of the needle; Q = Volume flow rate of the saline solution through the needle; η = Viscosity of the saline solution: ΔP = Pressure difference across the needle, write down an expression for ΔP in terms of r, l Q and η, when the needle is placed horizontally.

ii) When a needle with $r = 2 \times 10^{-4}$ m and $l = 3 \times 10^{-2}$ m is used, the volume flow rate through the needle, before it is inserted into the patient is $Q = 1.5 \times 10^{-7} m^3 s^{-1}$. Calculate the height h shown in figure (1) under these conditions. You are also provided with the following data.

Density of the saline solution = 1.2×10^3 kg m^{-3}, $\eta = 2 \times 10^{-3}$ Pas. Take $\pi = 3.0$

iii) If it is desired to maintain the initial volume flow rate through the needle at the same value given in (a) (ii) above, after inserting the needle into a place where the venous blood pressure of the patient is 3×10^3 Nm^{-2} over and above the atmospheric pressure, by how much the height h must be increased?

iv) If the length of the saline bottle is 0.2 m, by how much the volume flow rate through the needle will change when a completely filled saline bottle becomes almost empty?

v) Hence, find the average value of the volume flow rate through the needle.

vi) If a saline bottle contains 1.104×10^{-3} m^3 of saline solution, using the result obtained in (a) (v) above, find the time taken for the infusion of one bottle of saline completely to the patient.

b) Infusion under the gravity is not a very good method when it is crucial to maintain a constant rate of infusion. In this situation use of infusion machines is more appropriate. A schematic diagram of the relevant section of such an infusion machine is shown in the figure (2).

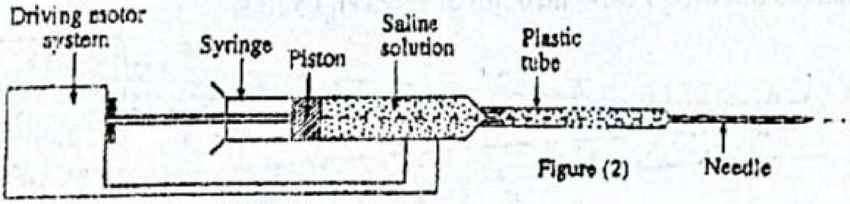


Figure (2)

Here the fluid is filled to a syringe and is pressed using a piston which can be moved very slowly by a controllable motor system. Consider that the needle described in (a) (ii) above is connected to this machine horizontally as shown in the figure. The machine is used to infuse the saline solution to the patient as described in (a) (iii) above with the same volume flow rate of $Q = 1.5 \times 10^{-7}$ m^3s^{-1}.

i) If the internal cross – sectional area of the syringe is 1.2×10^{-3} m^2, how fast the piston must be moved?

ii) Assuming that the pressure differences of the saline solution across the syringe and the plastic tube [see figure (2)] are negligibly small calculate the constant force exerted by the piston on the saline solution.

iii) Calculate the rate of work done by the driving motor system on the piston.

9) Write down Poiseuille's equation for the rate of flow, Q, of a liquid through a horizontal cylindrical narrow tube under a pressure difference of ΔP. Identify all the other symbols you used.

Under the above condition, the resistance exerted by the tube against the rate of flow of the liquid, Q, can be defined as the flow resistance $R = \frac{\Delta P}{Q}$.

a) What physical quantities associated with the tube and the liquid determine the flow resistance R?

b) When a liquid flows under pressure differences of ΔP_1, ΔP_2 and ΔP_3 through three horizontal narrow tubes connected in series as shown in figure (1), the flow resistances exerted by the tubes are R_1, R_2 and R_3 respectively. Using the definition given above for R show that the flow resistance, R_0, of the system can be written as $R_0 = R_1 + R_2 + R_3$. (Neglected edge effects).

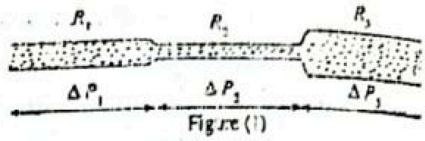


Figure (1)

Figure (2)

c) When a liquid flows through two horizontal narrow tubes connected in parallel under a common pressure difference ΔP as shown in figure (2), the flow resistances exerted by the tubes are R_1 and R_2. Show that the flow resistance R_0 of the system can be written as $\frac{1}{R_0} = \frac{1}{R_1} + \frac{1}{R_2}$ (Neglect end effects)

d) Figure (3) shows a set of horizontal narrow tubes A, B, C, D and E connected between the point X and a common reservoir Y so that a liquid can flow from X to Y. The pressures at X and Y are maintained at constant values. The flow resistance of each tube is labeled in the diagram in units of $mmHg\ s/cm^3$. Tube B is divided into two tubes C and D of equal flow resistances. This simplified model may also be used to illustrate the blood flow through arteries and veins.

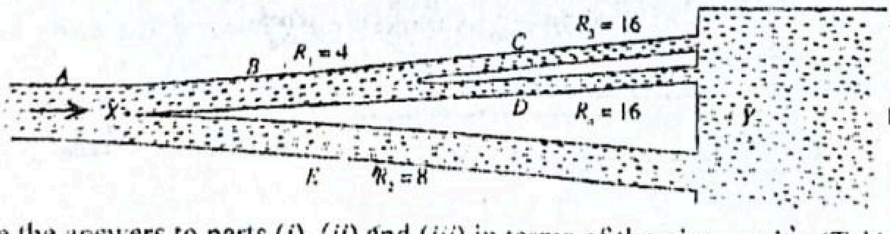


Figure (3)

Give the answers to parts (*i*), (*ii*) and (*iii*) in terms of the given units. (Take $\pi = 3$)

i) 1) Calculate the flow resistance, due to the system of tubes B, C and D between the points X and Y.

2) Calculate the flow resistance, due to the system of tubes, B, C, D and E between the points X and Y.

ii) If the flow rate of the liquid across X is 6 cm^3/s, calculate the pressure difference between X and Y.

iii) Using the above results, find the flow rate of the liquid through tube E.

iv) If the length of tube E is 2 cm find the internal radius of tube E. The viscosity of the liquid is 4.0×10^{-3} Pa s (Take 1 mmHg = 133 Pa)

e) If the temperature of one of the tubes in the system given in part (d) is reduce, explain what would happen to the flow rate of the liquid in that tube. Neglect the changes in radius and length of the tube.

2019 A/L

10) When an object is falling through a viscous medium, it is subjected to the buoyant force and the drag force. The buoyant force pushes the object upward while the drag force acts against the motion of the object with respect to the medium.

(a) The drag force for a solid spherical object falling in a liquid medium can be expressed by the Stokes' law.

(i) Write down the Stokes' formula for a solid spherical object and name the parameters.

(ii) Write down two assumptions that are used in deriving the Stokes' formula.

(b) Consider an air bubble rising gradually upward in a viscous fluid. Stokes' Law can be applied to determine the time taken by an air bubble to reach the surface of the fluid. Neglecting the effect of the pressure change with height, the instantaneous velocity $V(t)$ of an air bubble in a viscous medium at a given time t can be given by

$V(t) = V_T\left(1 - e^{-\frac{t}{\tau}}\right)$, where V_T, and τ are the terminal velocity and the relaxation time of the motion of the air bubble, respectively

(i) If the relaxation time for the motion of an air bubble in a viscous medium is 4 μs. calculate the time it takes for the instantaneous velocity to be 50% of V_T, from the rest (Take $\ln 0.5 = -0.7$)

(ii) Calculate the time taken by the air bubble to increase the instantaneous velocity from 50% to 90% of V_T. (Take $\ln 0.1 = -2.3$)

(iii) Considering the answers obtained in (b) (i) and (b) (ii) above, plot the variation of the instantaneous velocity of the air bubble as a function of time. Clearly indicate V_T on the graph.

(c) Consider an air bubble rising from the bottom of an oil tank which is filled upto 10 m height.

(i) Obtain an expression for the resultant force acting on the air bubble in terms of η, ρ_0, ρ_a, a and v. where η is the coefficient of viscosity of oil, ρ_0 is the density of the oil, ρ_a is the density of air, a is the radius of the air bubble, and v is the velocity of the air bubble.

(ii) It is given that $\eta = 7.5 \times 10^{-2}$ Pa s, $\rho_0 = 900$ kg m^{-3}, $\rho_a = 1.225$ kg m^{-3}, and the average radius of an air bubble a = 0.1 mm. Neglecting the weight of the air bubble, and the effect due to the variation of pressure with height, calculate the terminal velocity of the air bubble.

(iii) Calculate the radius of the air bubble just below the surface of the oil, if the internal pressure of the bubble is 100.33. kPa, atmospheric pressure is 100 kPa, and the surface tension of oil is 2.0×10^{-2} N m^{-1}.

(iv) Considering the change in radius of the air bubble with height, sketch the variation of its instantaneous velocity with time.